# CRANFIELD UNIVERSITY

## SILSOE COLLEGE

Faculty of Agricultural Engineering, Food Production and Rural Land Use

by

Luiz Henrique Antunes Rodrigues

# CAP-DAIRY: Computer Aided Planning of Dairy Farms

Supervised by Mr. Eric Audsley (Silsoe Research Institute)
and Mr. Duncan Kerr (Silsoe College)

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# ABSTRACT

A linear programming (LP) model has been developed (CAP-DAIRY) to describe the grass utilisation and feeding system on a dairy farm. It links several components of the system and optimizes the system as a whole. The model links a grass utilisation model, a feed ration model and a novel model which relates on-going milk yield to on-going feeding level.

The main feature of the model is the approach adopted to relate feeding levels, milk yield and weight changes. When cows are fed more than they require for maintenance and the current level of milk yield, the excess energy becomes increased bodyweight and cause an increase in milk yield. When fed less than they require, they mobilize reserves into energy for milk production and lose weight and tend to reduce milk yield. At the start of the lactation some weight loss is tolerated. This is treated in the model as a requirements for up to 0.5 kg/day weight loss in addition to maintenance so that a lower weight loss is the increase case. A linear mathematical model that represents this mechanism was developed and incorporated into the LP and fitted to data which changed the level of feeding of dairy cows during the lactation. This gives a greater flexibility to the LP and allows the model to determine optimal feeding levels at all stages of the lactation and as a consequence optimal milk yields and optimal stages for weight changes, which vary depending on calving date and feed availability

The grass utilisation model permits the successive utilisation of grass for grazing or silage making. Grazing can take place two, three or four weeks after the previous use and silage making five, six or seven weeks after the previous use. To allow for the effect of silage making on regrowth, use after this is delayed by one week. Data on energy and dry matter yields at any time is required and the model determines the optimum schedule of use and frequency. Silage is made in a number of separate silos but the feeding-out energy value makes the model non-linear. This is solved by using a recursive approach in which the initially unknown feeding value is calculated from successive solutions and the model re-optimized to convergence.

The feed ration model determines the amount of grass, silage and concentrates required based on the maximum dry matter intake, which is a function of yield and the energy required for maintenance, milk yield and any weight change. The model could be easily extended to also use protein given suitable data.

The LP determines the optimal land use for forage and cash crops, calving pattern and feeding strategy according to specific farm conditions such as farm area, milk quota and availability of forage maize. Several scenarios were studied and the effects of changes of different parameters analysed.

Results indicated that net margins increased with maize crop areas and gave higher optimum milk yields replacing concentrates up to an optimum area of maize.

The seasonality of milk prices affected particularly calving pattern and milk yield and the results suggested they led to more even milk production due to encouraging Autumn calving.

Results also showed that the optimal feeding levels is different for cows calving in different periods of the year resulting in different weight change pattern and milk yields. Spring calving cows lost more weight than cows calving in any other period, but regained the weight lost quickly. They also produced the lowest level of milk. Autumns calving cows had the highest milk yield and the lowest weight losses, although a longer period to regain that weight was optimal. Summer calving cows produced slightly less milk and lost slightly more weight than Autumn cows.

Another important aspect that results showed was the influence that maize silage has on farm decisions. The larger the maize crop area the higher the marginal price of milk quota, showing that milk quota constraint was more severe for those cases and consequently higher prices for extra milk quota could be paid.

CAP-DAIRY is suitable for analysing the impact that changes such as milk prices, fertilizer prices or concentrates prices would cause on the optimal plans. The model is also helpful to evaluate research priorities by analysing the effects caused by biological and technical changes such as grass varieties and machinery.

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# Chapter One

## 1. Introduction

There are some key areas of the dairy farm business where efficient management is very important and essential to assure profitability. The efficient use of the seasonal grass growth (concerning yield and digestibility) for grazing and conservation is one of them. Feeding the right feed at right times is another. There are many other important decisions that dairy farmers have to make according to their farm conditions. These decisions vary from determining the herd size and calving pattern to the method of conservation and adequate feeding level, which determines milk production.

It is well known that feeding costs are an important element of milk production. Since milk quotas have been introduced in the UK by the European Community (now European Union), in 1984, levels of milk production, and therefore the milk receipts, have been limited. Consequently, farmers' interests have moved from expanding their herd size and milk production to producing milk at the lowest possible cost. Milk quota, however, can be purchased or leased and prices of milk quota are determined by the market. So dairy farmers have to consider the return they could get from increasing their milk quota.

There is a strong relationship between the reduction of feed costs and forage area management. Seasonality of grass growth has a crucial role and its optimal use determines both the optimal grass area and optimal sequence of use of the land.

Another important component to be considered in any dairy farm system is the efficiency of energy use by dairy cows during the lactation and during the dry period. Cows fed at high and low level gain and lose weight, respectively, and these weight changes directly affect their milk yield. The optimal feeding strategy and forage area management must take account of the feeding levels as they affect weight changes and milk yield at subsequent stages.

Dairy farming is a business like any other and the primary objective is profit, therefore the higher prices for Summer milk and lower prices for Spring milk are important considerations. It does not make sense for a farmer to feed cows to meet their requirements if there is no benefit. A view of the whole system (in all stages of the lactation throughout the year) might indicate some situations where it would be worth feeding dairy cows at lower levels in early or mid lactation, when the requirements are higher due to the peak milk yield, and increase the feeding level later, in order to reduce the drop in milk yield. Obviously, biological limitations must be observed, for they constrain the capacity of cows to reverse weight losses.

The impact of weight loss and gain on the annual profit is an important topic and strategic decisions that take feeding levels into account and their effects on weight changes and on milk yield are more likely to increase profits.

Another important aspect to be considered is that dairy farmers need to make the best use of their land, since they are constrained by milk quota. High intensity feeding may be correct at some time of the year, but not at others due to, for example, Summer grass or silage quality. Furthermore, depending on calving time different levels of feeding may be appropriate. This means that dairy farmers have to consider whether it is better to use some area for non dairying enterprises.

Many linear programming (LP) models have been developed to formulate feed rations. Most of them are for given daily milk yield targets, however alternative production levels with potentially higher incomes have not been analysed and compared. Other models take the whole system into account, but assume fixed lactation curve and weight changes irrespective of available forage.

In this thesis, an LP model is developed (CAP-DAIRY) to describe the whole dairy farm system which incorporates another model describing the relationship between levels of feeding and weight loss or gain and milk production. The latter takes account of the weight loss as a nutrient resource, especially to meet high energy requirements, unlike most existing models which assume that requirements are met entirely by the ration. The LP model assures that solutions for any stage of lactation do not ignore information from previous and subsequent stages and takes into account the interdependence within several stages of the calving interval.

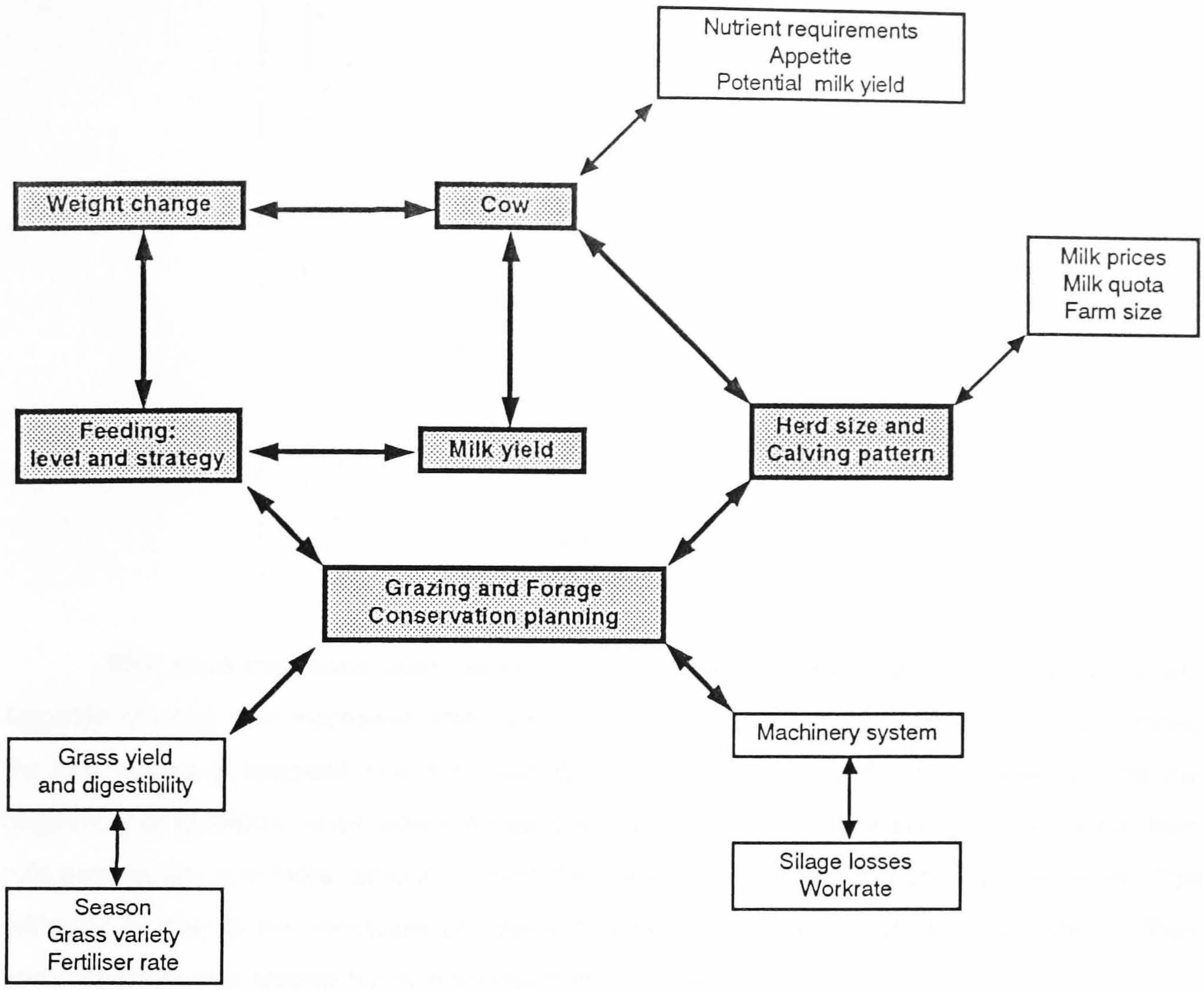
The LP model optimizes the herd size, the calving pattern, feeding strategy and forage area management according to specific farm conditions such as size, milk quota and conservation method. The optimal forage area management is provided not only as the optimal grass area for grazing and conservation, but it also indicates the optimal sequence of grazing and silage-making over the season.

An important point to highlight is that the model aims to indicate optimal stages for weight loss and gain rather than determine the precise loss or gain.

The model can be used to examine new machinery and silage making alternatives, the effects of economic changes (e.g., milk and concentrate prices) and the impact of new crops such as maize silage on the farm plan.

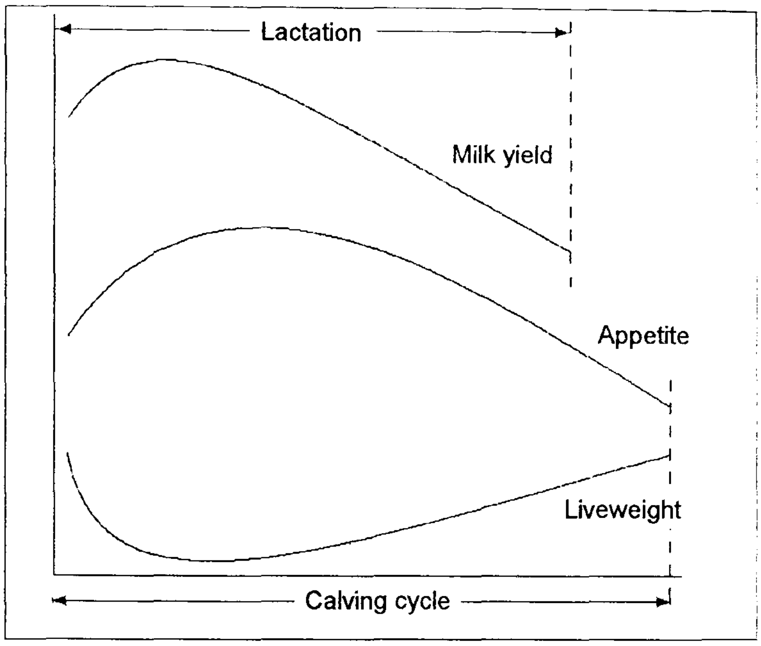
Figure 1.1 summarises the whole system and shows the strong relationship between its several elements. Darker boxes represent decisions that must be made and white boxes represent technical and economic parameters.

Figure 1.1 - Dairy farm system




The relationship between weight changes and milk yield is strongly affected by the cow's appetite (intake capacity). Figure 1.2 illustrates the three elements throughout the calving cycle: lactation curve, appetite and liveweight.

**Figure 1.2 - Relationship between milk yield, appetite and liveweight**




Milk yield increases after parturition and achieves a peak at around the 5th week. Appetite of cows also increases after parturition, although a marked reduction occurs during the first weeks of lactation and the peak is achieved at around the 10th week. During the beginning of lactation, when cows demand a substantial amount of energy to increase their milk production, it is more difficult to feed them and provide them the energy they need. This difficulty is due to the limitation of intake. During this period, cows usually mobilize their body reserves into energy for milk production and lose weight.

The level cows are fed during the beginning of lactation will determine the milk yield and weight change for the remainder of the lactation.

# Chapter Two

## 2. Literature Review

Models are useful tools to help the understanding of farm systems because they can represent different system components, their interactions, inputs and outputs. Spedding (1988) states the importance of analysing any system as a "whole system" in which changes can only be considered improvements if they result in improvements of the system as a whole.

Mathematical models are especially suitable for complex systems like dairy farms because they can gather information from several components and provide a holistic view of the system (France and Thornley, 1984). Optimization models are particularly useful to highlight areas of research where further developments might be worthwhile.

Dairy farmers constantly make management decisions that affect the profit of their farm business. These decisions may concern the herd (e.g., calving pattern and herd size), milk production (e.g., the level of supplementary feeding and acquisition of milk quota) or forage conservation (e.g., area to be conserved, harvesting time and method of conservation). Any decision involving any component affects the whole system.

Several models have been developed for the formulation of feed rations which are useful to calculate the least cost ration formulation for dairy cows at a certain stage. However, they are not suitable for any long term plan because they are usually "static" and do not take into account either the whole system or the whole year. Hulme et al (1986) developed a bio-mathematical model (CAMDAIRY) that incorporates several functions to predict nutritional requirements and feed intake of a lactating cow. Two main features of this model are the inclusion of the phenomenon of nutrient partition and the reduction of forage intake when fed with concentrates, called "substitution effect". The partition of nutrients takes into account the non-linearity of the relationship between energy intake above maintenance and milk production. As energy intake above maintenance increases, milk production response also increases, but with a decreasing rate, due to partition of nutrients from milk to body tissue. Despite the complexity of the model and its accuracy to predict intake and nutritional requirements of lactating cows, it has a limited application to be used in management models due to the large amount of detailed data it requires. Another limitation of this model is that it does not take into account the relationships between different stages of lactation.

Olney and Standing (1989) developed a model to calculate the most profitable rations for cows (DAIRYFEED). This model takes account of the nutritional requirements of cows according to the stage of lactation, and then calculates the most profitable milk yield, according to the availability and prices of feeds, milk price at the particular time it is being calculated and milk quota of the farm. Its purpose is to determine the milk production that will maximize profit at a particular time of the year and formulate the cheapest ration to achieve that production level. This model has also a limited application as it neither takes into account the whole lactation period nor the effects of one particular stage of lactation on the subsequent lactation stages. One positive feature of this model is the inclusion of prices of both feed and milk, when formulating the ration.

Forage has an important role on dairy farms and several models have been developed to optimize land use for this purpose (i.e., for grazing or for conservation).

Audsley (1974) developed a linear programme (LP) model that determines the optimal grass cutting schedule, incorporating the seasonality of the grass growth rate. The model deals with a grass-drying enterprise and its objective is to determine the optimal sequence to harvest the grass to be dried, in order to maximize the total return less costs over the season. The model does not include livestock, but a detailed description of the constraints of the LP model is given. With a few changes, this approach can be included in a more general model, determining not only the optimal areas for grazing and silage-making, but also the optimal sequence of both activities.

Dumont and Boyce (1976) developed an LP model to study the benefits of including an alternative system to produce silage in existing farm systems. Their LP model was based on the LP model previously developed by Audsley (1974), but included the livestock requirements, compared a traditional system for the production of silage with an alternative system (forage fractionation) and showed the effects of the latter on the gross margin of the farm. It determined the optimal combination of grass area for grazing and silage-making, herd size and purchased feed to give the maximum profit. The model was limited to follow milk production and weight change patterns recommended by the Ministry of Agriculture, Fisheries and Food (MAFF). It did not take into account the effects of weight loss or gain in one stage on milk yield in subsequent stages; hence, alternative feeding levels, with consequent different milk yields, could not be compared.

Pichard et al (1989) also developed an LP model to find optimal land use for forage, taking into account the seasonality of the grass growth rate and its nutritional quality. The model incorporates the nutritional requirement variation, due to physiological stages of the animal. It also considers pre-determined combinations of crops and conservation strategies (e.g., 1 , 2 or 3 cuts), and optimizes the area to be allocated for each one. It does not however determine the optimal sequence of use of the land.

Several general dairy farm models, that consider the whole system, have also been developed.

Doyle and Edwards (1986) developed a simulation model to evaluate economic consequences of changes in the forage area management (grazing and silage-making) and in the feeding level. One of the main features of the model is that it allows a previously selected grassland management regime to be re-evaluated as the season progresses. Unpredicted changes in grass yield, due to uncertainty of the weather forecast, can be added to the problem in a later stage and appropriate changes to land use may be made from that point onwards. It is a simulation model and does not determine the optimum strategy for the system, although the user can compare different scenarios and choose the most suitable one for particular farm conditions. The model also assumes pre-determined weight changes and milk production patterns and does not consider the effects of changes in early stages on subsequent stages.

Olney and Falconer (1985) developed an LP model that allows for several feed sources and takes seasonal milk prices into account, but does not calculate the optimal schedule for grass cutting and grazing. It has proved useful to highlight other aspects, such as improvement of grass species with early maturation, which has been shown to have a strong influence on the calving pattern and can increase the farm profit. It assumes pre-determined milk yield levels and liveweight change pattern.

Olney and Kirk (1989) presented an LP model that represents a dairy farm system and determines management strategies to maximize profit. The model optimizes the herd structure (calving pattern and herd size), taking into account the nutritional requirement and availability of pasture. It also determines the optimal feeding level of concentrates, and the land use for conservation. The model can be used to find the most profitable management plan. One of its limitations is that it does not optimize the milk production level. It finds the optimal plan to achieve a pre-determined milk yield and liveweight change pattern. Although the model takes into account the availability of land for pasture and conservation, it optimizes only the area required and not the strategic sequence of use.

Goss (1987) presented an LP model that optimizes milk production and feeding levels of concentrates, and areas for grazing and silage-making, but it is limited to optimize these factors for a pre-determined calving-pattern (e.g., spring calving or autumn calving), assuming pre-determined silage-making dates. It also assumes pre-determined milk yield levels and liveweight change patterns and, although it optimizes the grazing area, it does not determine the optimal grazing sequence. The thesis contains an extensive literature review of important topics concerning dairy farms and milk production.

Hansen (1990) developed a Decision Support System (DSS) model to optimize land use and feed supply on dairy farms. Fodder crop production, feed storage and milk production are considered in the model. One of the main features of the model is to provide a range of quasi-optimal solutions (i.e., solutions where the value of the objective function is just a small percentage worse than the optimal solution). These alternatives allow the farmer to select the best plan according to personal preferences.

Killen and Kleane (1978) developed an LP model to determine a calving pattern that minimizes milk production costs and which takes into account the seasonality of both grass growth rate and milk prices. The dual solution of the problem is used to determine the seasonal prices that should be paid to milk producers to reflect the seasonal aspects of production costs. The model does not optimize the milk yield level and follows a pre-determined liveweight change pattern.

One dairy farm management model which took into account the liveweight change as part of the strategic decision was the LP model developed by Reyes et al (1981). The model optimizes the milk yield level, taking into account the effects of the feeding levels on the liveweight change and on the milk production. It is a multistage model that represents the interdependence between stages of lactation and the dry period. It examines the whole system and finds the best strategy of weight change and milk production. It determines optimal grazing areas, but does not optimize the sequence of use of the land throughout the season and assumes that land used for grazing pasture at any stage cannot be used for anything else in subsequent stages. It does not consider silage-making at all, so this limits the decision to either allow dairy cows to graze or be fed with purchased feed.

When optimizing a whole dairy farm system throughout the year, nutritional requirements of cows have an important role, as the responses over the whole lactation are greater than short-term responses. This indicates a cumulative effect from early and late lactation and the dry period. During the first weeks of lactation, cows have a lower appetite and consequently it is difficult to meet their energy requirements. Body reserves are then mobilized during this period to supply energy for milk production and subsequently cows lose weight (NRC, 1988; MAFF, 1984; France et al., 1982).

Several experiments report that cows should receive diets with higher energy levels in early lactation to prevent excessive weight losses, which might prove difficult to regain later, with subsequent reduction in milk yields (Poole, 1986; Broster et al, 1975; France et al, 1982). Although many experiments have shown that body weight and milk yield are affected by the energy level of the diet, few investigations have been made concerning the relationship between body weight changes and milk production (Wood et al, 1980). Broster et al (1993) state that few full lactation studies have been undertaken, despite the strong influence of the early lactation on the remaining period.

# Chapter Three

## 3. Liveweight change effects on milk yield

Dairy cows require energy for maintenance, milk production, pregnancy and growth. During their first weeks of lactation it is recognised that they have a lower appetite and consequently their energy intake is not enough to meet their requirements. However, they have reserves of energy to carry over those periods of low energy intake. The energy they give as milk during the first weeks of lactation they have generally previously stored as fat during pregnancy. Body reserves are then mobilized and supply energy for milk production and hence they usually lose weight during this period (NRC, 1988; MAFF, 1984; France et al, 1982). It is well known that low energy intake over a period causes the lactating animal to reduce its milk yield and this can never be fully recovered.

Few studies have been done taking the genuine full lactation into account (Broster et al, 1993). Even research that has taken account of the "full lactation" has avoided the first few weeks post-calving, when several problems arise due to the quick changes in milk composition, milk yield, feed intake and perhaps health problems. Complications of late lactation, such as low milk yield and influence of pregnancy, are also often avoided

A mathematical model describing the bioenergetic system of lactating and pregnant cows is proposed to represent the relationship between energy intake, milk production and liveweight changes throughout the lactation. The model takes into account the residual effects on subsequent periods. The model is based on MAFF recommendations (MAFF, 1984) and includes a mechanism to balance the energy for growth, for maintenance and for milk production throughout the lactation.

### 3.1. Notation for the mathematical model describing the relationship between liveweight changes and milk production

$E_f$ : Energy fed (MJ/day)
$E_m$ : Energy for maintenance (MJ/day)
$E_p$ : Energy for pregnancy (MJ/day)
$E_w^+$ : Energy for weight gain (MJ/kg gain)
$E_w^-$ : Energy from weight loss (body reserves) (MJ/kg loss)
$E_y$ : Energy for milk production (MJ/kg milk)
$\Delta E^+$ : Surplus energy (MJ/day)
$\Delta E^-$ : Deficit energy (MJ/day)
$I$ : Expected dry matter intake (based on the expected milk yield) (kgDM/day)
DM : Actual dry matter intake (kgDM/day)
$\Delta w^+$ : Weight gain (kg/day)
$\Delta w^-$ : Weight loss (kg/day)

Y : Expected milk yield (kg/day)
y : Actual milk yield (kg/day)
f : factor that relates dry matter intake and milk yield
$\Delta y^{+}$ : Milk yield increase (kg/day)
$\Delta y^{-}$ : Milk decrease (kg/day)
$\alpha^{+}$ : proportion of the surplus energy allocated to milk production
$\alpha^{-}$ : proportion of the deficit energy allocated to milk production
$\gamma^{i}$ : perpetual effect of weight change on milk yield in subsequent periods

## 3.2. Description of the mathematical model relating liveweight changes and milk production

### 3.2.1. Energy balance

The total energy intake is allocated to cow's maintenance, pregnancy and milk production. Any excess of energy is partitioned between milk production (milk yield increase) and body reserves (weight gain). A mobilization of body reserves occurs when there is a deficit of energy, being partitioned between yield loss and weight loss. The energy balance is described in equation ( 3.1).

$$E_f = E_m + E_p + E_y Y + \left[E_w^{+} \Delta w_{+} - E_w^{-} \Delta w^{-}\right] + \left[E_y \Delta y^{+} - E_y \Delta y^{-}\right] \tag{3.1}$$

It is assumed that cows have a potential milk yield (Y), and when they are fed the exact amount of energy required for maintenance, pregnancy and to produce the potential milk yield, they do not gain or lose weight and produce the expected milk yield. The lactation curve for these potential yields throughout the lactation is calculated on the basis of Wood's model (1967) to predict milk yield.

When cows are overfed, there is a surplus energy $\Delta E^{+}$, which is the energy fed above the requirements for maintenance, pregnancy and the potential milk production. It is described in equation (3.2).

$$\Delta E^{+} = E_f - E_m - E_p - E_y Y \tag{3.2}$$

A proportion $\alpha^{+}$ of the surplus energy can be allocated to milk production, increasing the yield is shown in equation (3.3)

$$E_y \Delta y^+ = \alpha^+ \Delta E^+ \tag{3.3}$$

and the remainder of the surplus energy is allocated to weight gain, as shown in equation (3.4)

$$E_w^+ \Delta w^+ = (1 - \alpha^+) \Delta E^+ \tag{3.4}$$

Rearranging equation (3.4) we have equation (3.5)

$$\Delta E^+ = \frac{E_w^+}{(1 - \alpha^+)} \Delta w^+ \tag{3.5}$$

From equations (3.3) and (3.5), we have equation (3.6), which is convenient because it directly relates weight gain and milk yield.

$$\Delta y^+ = \frac{\alpha^+}{(1 - \alpha^+)} \frac{E_w^+}{E_y} \Delta w^+ \tag{3.6}$$

Similarly, when cows are underfed, there is a deficit energy $\Delta E^-$, described in equation (3.7).

$$\Delta E^- = E_m + E_p + E_y Y - E_f \tag{3.7}$$

A proportion $\alpha^-$ of the deficit energy is allocated to milk production and the milk yield decreases as shown in equation (3.8)

$$E_y \Delta y^- = \alpha^- \Delta E^- \tag{3.8}$$

and the remainder of the deficit energy affects the weight loss as shown in equation (3.9)

$$E_w^- \Delta w^- = (1 - \alpha^-) \Delta E^- \tag{3.9}$$

From equation (3.9) we derive equation (3.10).

$$\Delta E^{-} = \frac{E_{w}^{-}}{(1-\alpha^{-})}\Delta w^{-} \tag{3.10}$$

From equation (3.8) and (3.10) we have equation (3.11), which relates weight loss to milk yield.

$$\Delta y^{-} = \frac{\alpha^{-}}{(1-\alpha^{-})}\frac{E_{w}^{-}}{E_{y}}\Delta w^{-} \tag{3.11}$$

When cows are underfed, their body reserves are mobilized to supply energy for milk production; from equations (3.8) and (3.9), it can be seen that the lower the $\alpha^{-}$ the lower the yield decrease and the higher the weight loss. In other words, the more body reserves are mobilized to supply energy the higher the weight loss and the lower effects on milk yield.

In order to simplify the terms, we define

$$\lambda_{0}^{+} = \frac{\alpha^{+}}{(1-\alpha^{+})}\frac{E_{w}^{+}}{E_{y}} \quad \text{and} \quad \lambda_{0}^{-} = \frac{\alpha^{-}}{(1-\alpha^{-})}\frac{E_{w}^{-}}{E_{y}}$$

The effect of changes in any period affects the subsequent periods. It is assumed that this effect decreases at a weekly rate $\gamma$ ( $0 \le \gamma \le 1$ ).

$$\lambda_{i}^{+} = \gamma^{i}\lambda_{0}^{+} \quad \text{and} \quad \lambda_{i}^{-} = \gamma^{i}\lambda_{0}^{-}$$

## 3.2.2. Milk yield

The actual milk yield (equation (3.12)) is the expected milk yield corrected by the yield increase or decrease when cows are overfed or underfed, respectively.

$$y = Y + \Delta y^{+} - \Delta y^{-} \tag{3.12}$$

### 3.2.3. Appetite (Dry Matter Intake)

Similarly to the actual milk yield, the actual dry matter intake is the expected dry matter intake (equation (3.13)), which is calculated taking into account the liveweight and the expected milk yield, corrected by the increase or decrease of the milk yield (multiplied by a factor f ).

$$DM = I + f\Delta y^{+} - f\Delta y^{-} \tag{3.13}$$

### 3.2.4. Generic equations throughout the lactation in a weekly format

In a weekly format, the model is described in equations (3.14), (3.15) and (3.16).

<u>*Milk yield*</u>

The actual milk yield in week $j$ ($y_j$) is calculated from the expected milk yield in week $j$ ($Y_j$) plus the cumulative effects of weight changes since the beginning of the lactation.

$$y_j = Y_j + \sum_{k=1}^{j} \lambda^{+}_{(j-k)} \Delta w^{+}_{k} - \sum_{k=1}^{j} \lambda^{-}_{(j-k)} \Delta w^{-}_{k} \tag{3.14}$$

<u>*Appetite*</u>

The actual dry matter intake in week $j$ ($DM_j$) is calculated similarly to the actual milk yield, with the cumulative effects of weight change being summed to the expected dry matter intake in week $j$ ($I_j$).

$$DM_j = I_j + f\sum_{k=1}^{j} \lambda^{+}_{(j-k)} \Delta w^{+}_{k} - f\sum_{k=1}^{j} \lambda^{-}_{(j-k)} \Delta w^{-}_{k} \tag{3.15}$$

*Energy*

The energy fed in week $j$ ($E_{fj}$) is allocated for maintenance ($E_m$), pregnancy ($E_p$) and actual milk yield in week $j$ ($y_j$). If there is an excess of energy, the cow will gain weight that week ($\Delta w_j^+$). If there is a deficit, she will lose weight that week ($\Delta w_j^-$).

$$E_{fj} = E_m + E_p + E_w^+\left[1+\frac{\alpha^+}{(1-\alpha^+)}\right]\Delta w_j^+ - E_w^-\left[1+\frac{\alpha^-}{(1-\alpha^-)}\right]\Delta w_j^- + E_y\left[Y_j + \sum_{k=1}^{j-1}\lambda_{(j-k)}^+\Delta w_k^+ - \sum_{k=1}^{j-1}\lambda_{(j-k)}^-\Delta w_k^-\right] \quad (3.16)$$

## 3.3. Validation of the model proposed to describe the relationship between liveweight changes and milk production

In order to test the proposed model describing the relationship between liveweight changes and milk production, the same treatments and diets used in the experiment presented in Broster et al (1975) were simulated using this model. The values predicted for milk yield and weight changes were compared to the experimental data.

### 3.3.1. Description of the experiment by Broster et al (1975)

The experiment consisted of six treatments with three diets with different energy contents. Table 3.1shows the energy content of each diet and Table 3.2 summarizes the six treatments, showing the period of each diet for each treatment.

**Table 3.1 - Energy content of diets**

| Diet | Metabolisable Energy (MJ/day) |
|---|---|
| Very Generous (VH) | 176.8 |
| Generous (H) | 146.6 |
| Restricted (L) | 116.3 |

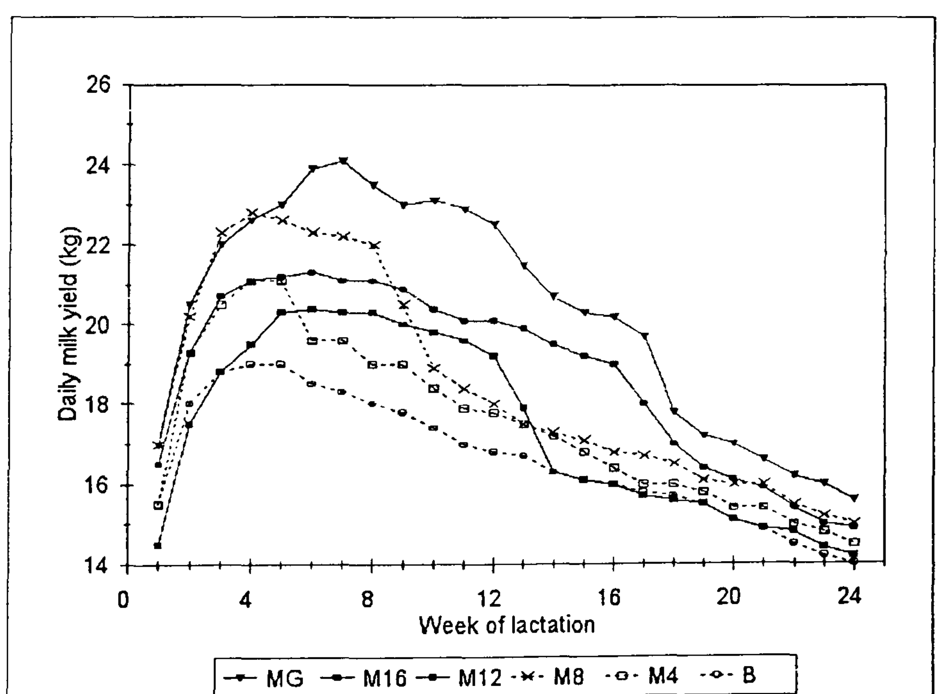
**Table 3.2 - Distribution of experimental diets over weeks 1-24 of lactation**

| | Weeks of lactation | | |
|---|---|---|---|
| Treatment | Generous (H) | Very generous (VH) | Restricted (L) |
| MG | 1-4, 13-16 | 5-12 | 17-24 |
| M16 | 1-16 | — | 17-24 |
| M12 | 1-12 | — | 13-24 |
| M8 | 1-8 | — | 9-24 |
| M4 | 1-4 | — | 5-24 |
| B | — | — | 1-24 |

## 3.3.2. Experimental results

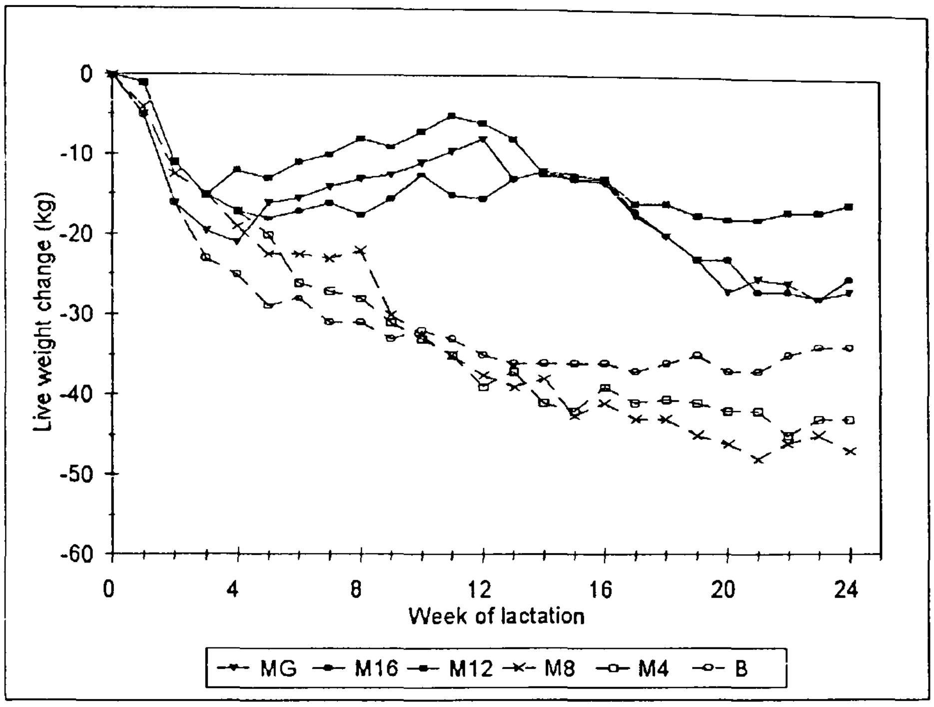
Milk yield and liveweight changes measured in the experiment are shown in Appendix I and plotted in Figure 3.1 (milk yield) and Figure 3.2 (liveweight changes). Figures were extracted from the graphs of the original paper by Broster et al (1975).

**Figure 3.1 - Milk yield (kg/day) over the first 24 weeks of lactation**




In Figure 3.1 one can be seen the strong effect of feeding levels during the beginning of the lactation on the milk yield in subsequent stages.

Figure 3.2 - Changes in liveweight (kg) over the first 24 weeks of lactation




Broster et al (1975) state that treatment M12 was anomalous, with a lower milk production than the other groups and the least liveweight loss over the 24-week period. No reason could be given to account for this.

### 3.3.3. Adjustment of the experimental results

In order to make the model comparable to the experimental results, experimental errors should be eliminated (or minimized). The results were adjusted by calculating the liveweight change so that the energy balanced for all weeks and for all treatments. Measuring milk yield is easier than weighing cows. It is also more reliable since it is less affected by factors such as time it occurs (before or after eating). For that reason, experimental milk yields were assumed to be correct, although with different energy contents, and liveweight changes were adjusted, according to the energy balance calculated weekly. The parameters used when calculating the energy balance were those recommended by MAFF (1984).

The energy intake is used for maintenance, milk production or weight change. Note that the period of the experiment is from week 1 to 24, when cows are not pregnant or are in the very beginning of pregnancy. Hence, energy for pregnancy is neglected and not taken into account when the energy balance equation (3.17) was calculated.

$$E_f = E_m + E_y y + E_w^+ \Delta w^+ - E_w^- \Delta w^- \qquad (3.17)$$

### 3.3.3.1. A statistical treatment to estimate the energy required for milk production

The parameter energy for milk production ($E_y$) shows the amount of energy required to produce a specific amount of milk (MJ/kg of milk). It measures the efficiency of the cows to convert energy into milk: the higher the $E_y$ the lower the efficiency (the cow needs more energy to produce the same amount of milk).

Parsons (1992b) and Hulme et al (1986) present the relationship between milk production and the energy above maintenance. The relationship is non linear and shows that the higher the energy above maintenance the lower the efficiency to convert it into milk (higher $E_y$).

Energy for milk production ($E_y$) of the experiment was estimated by linear regression of the experimental net energy ($E_{net}$) and milk yield.

Equation (3.18) determines net energy to produce milk ($E_{net}$), after energy for maintenance and weight gain are discounted. Energy from body reserves, when cows are losing weight, increases the net energy to produce milk.

$$E_{net} = E_f - E_m - E_w^+ \Delta w^+ + E_w^- \Delta w^- \tag{3.18}$$

Energy for maintenance ($E_m$) was calculated according to MAFF's recommendation (MAFF, 1984) and assumed to be constant throughout the period. The liveweight to calculate ($E_m$) was the average liveweight of the cows of the experiment: 540 kg ($E_m$= 57.44 MJ/day).

Equation (3.19) shows the relationship between net energy and milk yield.

$$E_{net} = E_y y \tag{3.19}$$

A linear regression of equation (3.19) for each treatment (MG, M16, M12, M8, M4 and B) showed no significant statistical differences for ($E_y$) in each treatment.

Another linear regression of equation (3.19) was repeated, this time for each energy level (VH, H and L) ignoring the treatment. The fitted parameters ($E_y$) and their standard errors are presented in Table 3.3.

**Table 3.3 - Estimated $E_y$ and standard errors**

| Energy level | $E_y$ | Standard error |
|---|---|---|
| L | 3.8488 | 0.0550 |
| H | 4.6534 | 0.1168 |
| VH | 4.7936 | 0.1029 |

Figure 3.3 to Figure 3.5 show the graphs of net energy versus milk yield for energy levels L, H and VH, respectively; the scattered points are the experimental points and the line is the fitted curve.

**Figure 3.3 - Linear regression - Restricted feeding level (L)**

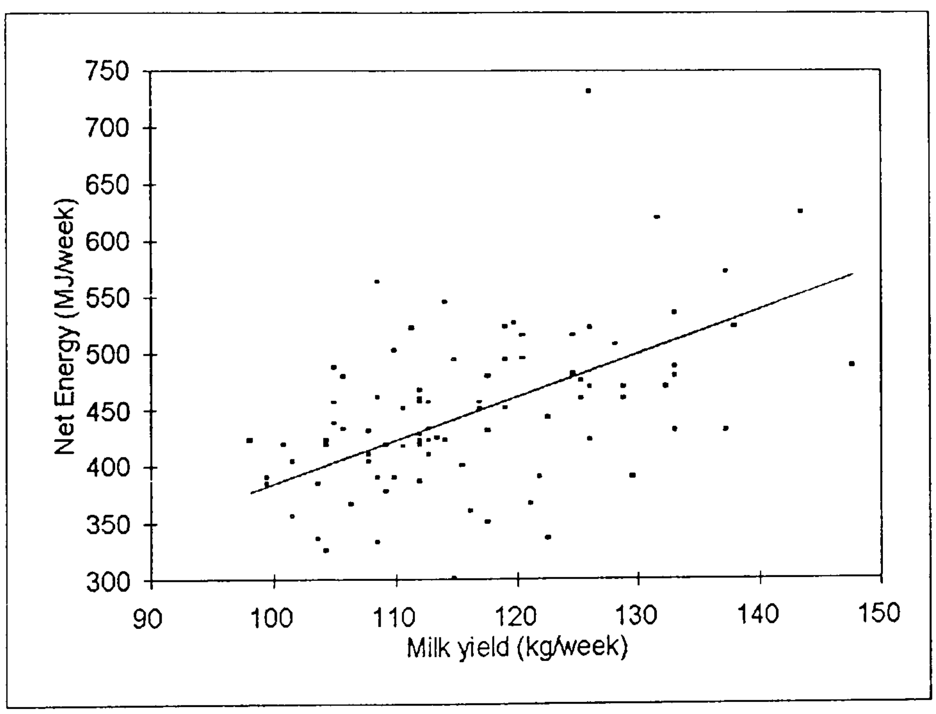



**Figure 3.4 - Linear regression - Generous feeding level (H)**

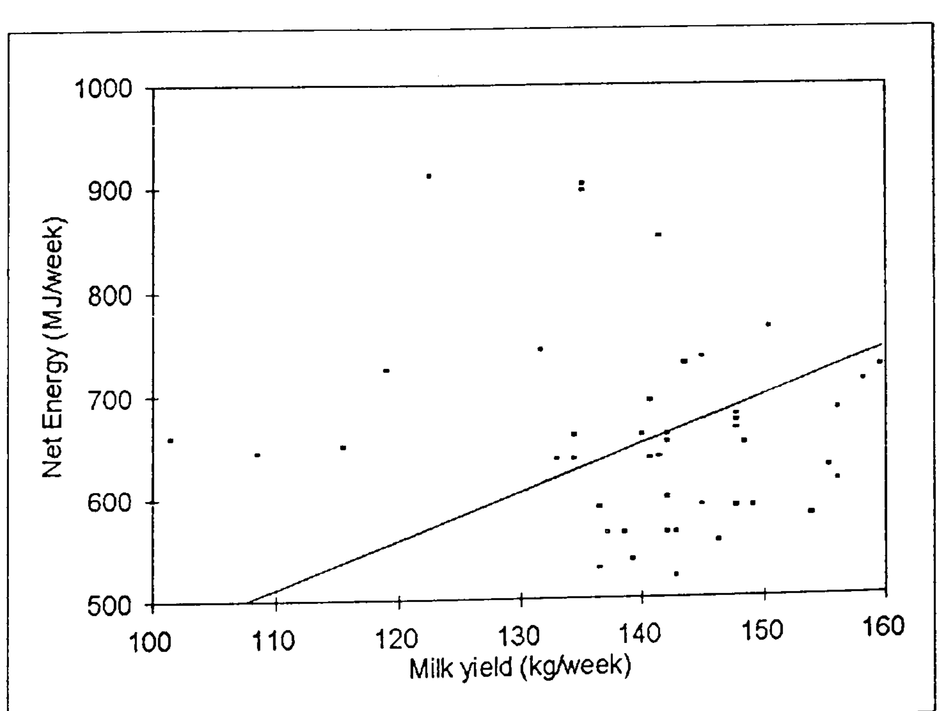
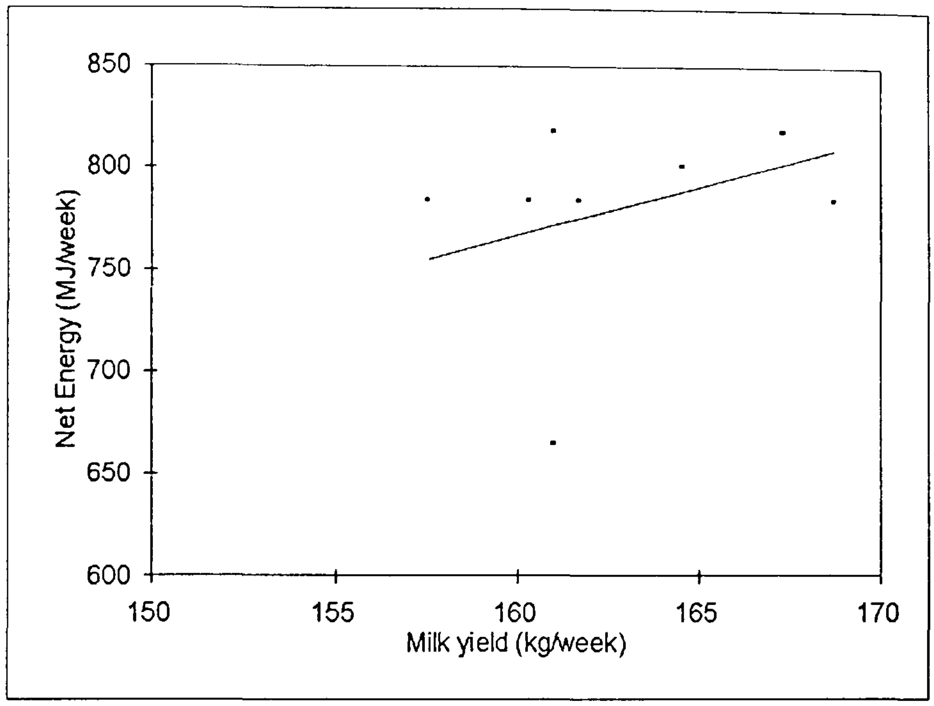


Figure 3.5 - Linear regression - Very generous feeding level (VH)




There is an agreement between these values and the expected behaviour for energy efficiency for milk production; the higher the energy level intake the lower the efficiency to convert this energy into milk (Parsons, 1992b; Hulme et al, 1986).

### 3.3.3.2. Adjusted experimental liveweight changes

Using ($E_y$) estimated for each level of energy intake, liveweight changes were "corrected" weekly using the energy balance described in equation (3.17).

Appendix II presents, in a weekly format, the energy fed, the milk yield measured in the experiment and the adjusted liveweight changes for each treatment.

Liveweight changes measured and adjusted are plotted in Figure 3.6 to Figure 3.11; treatments MG, M16, M12, M8, M4 and B, respectively.

**Figure 3.6 - Accumulated liveweight changes originally measured and adjusted (Treatment MG)**

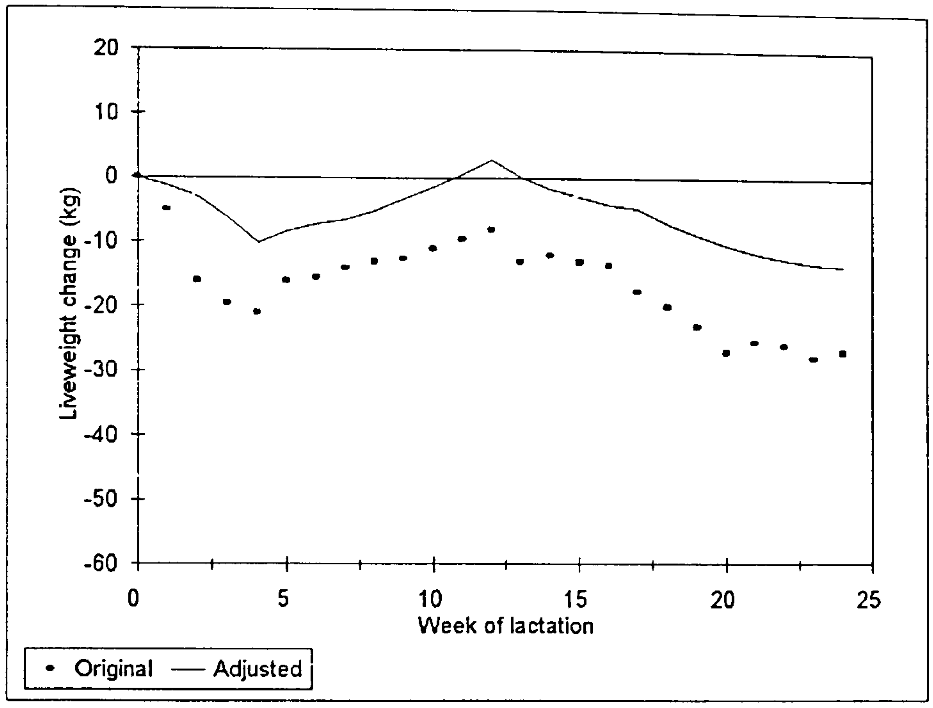



**Figure 3.7 - Accumulated liveweight changes originally measured and adjusted (Treatment M16)**

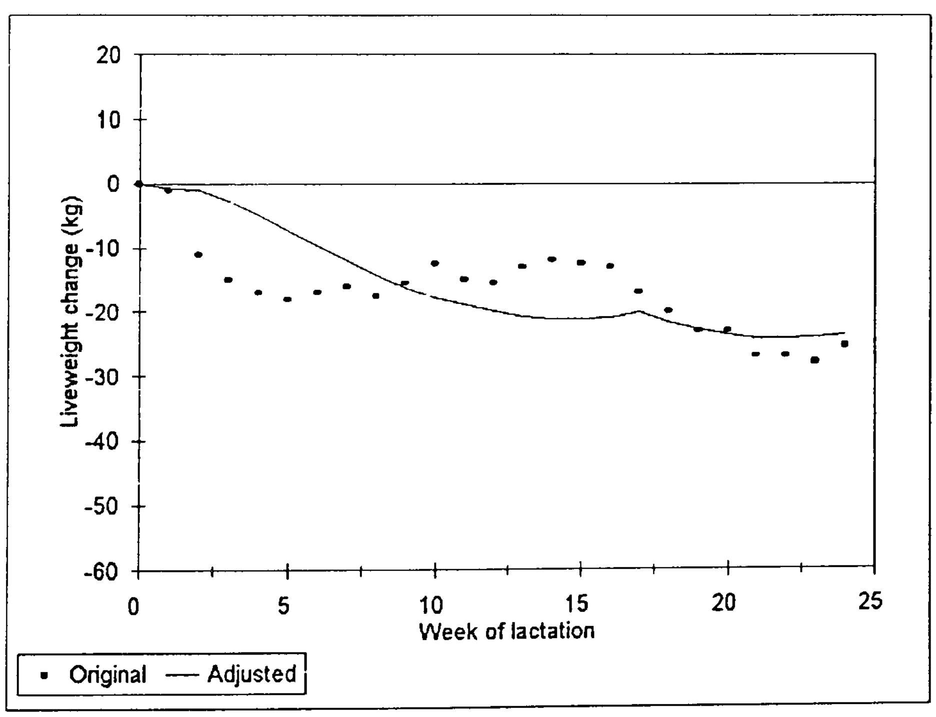


Figure 3.8 - Accumulated liveweight changes originally measured and adjusted (Treatment M12)

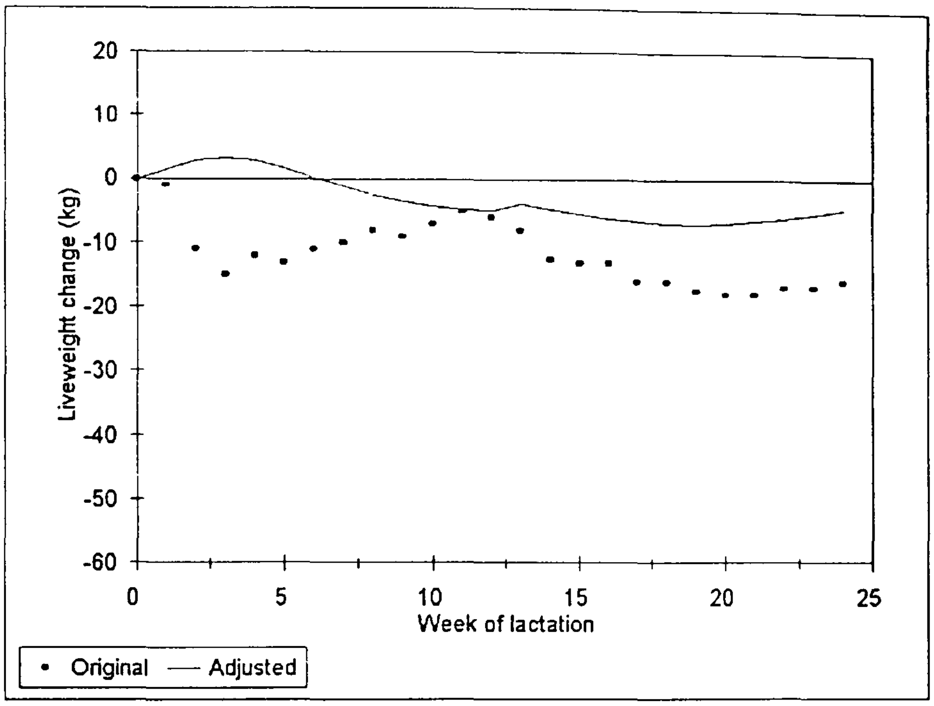



Figure 3.9 - Accumulated liveweight changes originally measured and adjusted (Treatment M8)

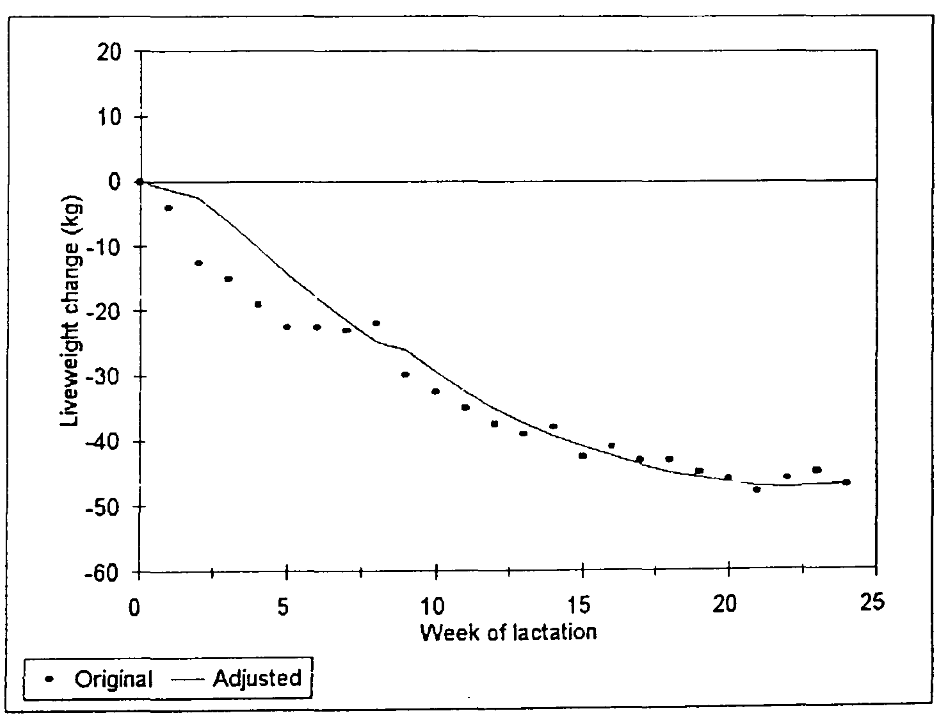


Figure 3.10 - Accumulated liveweight changes originally measured and adjusted (Treatment M4)

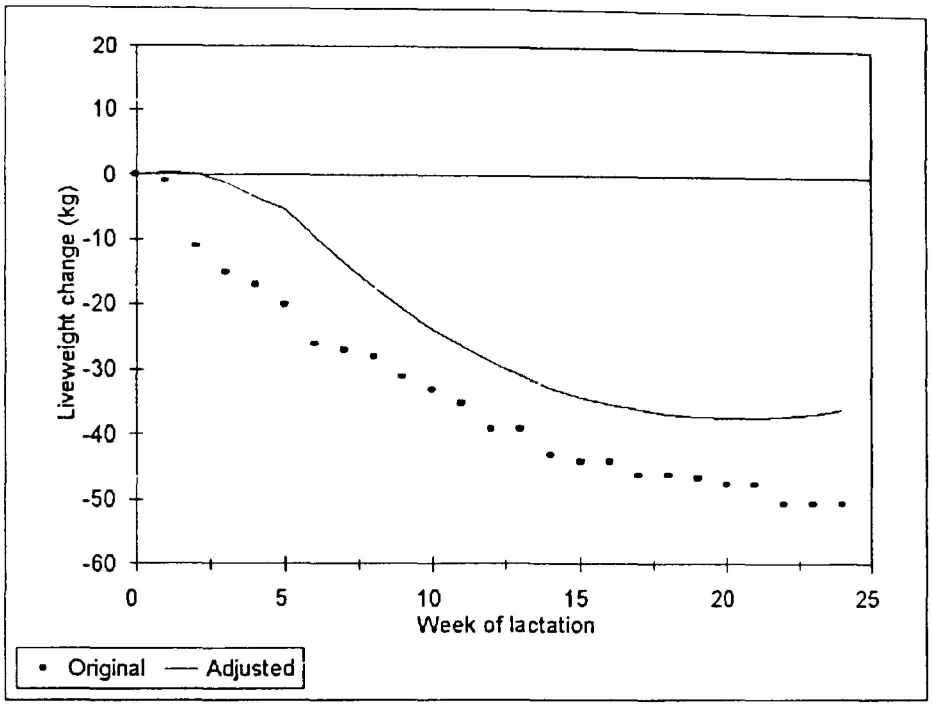
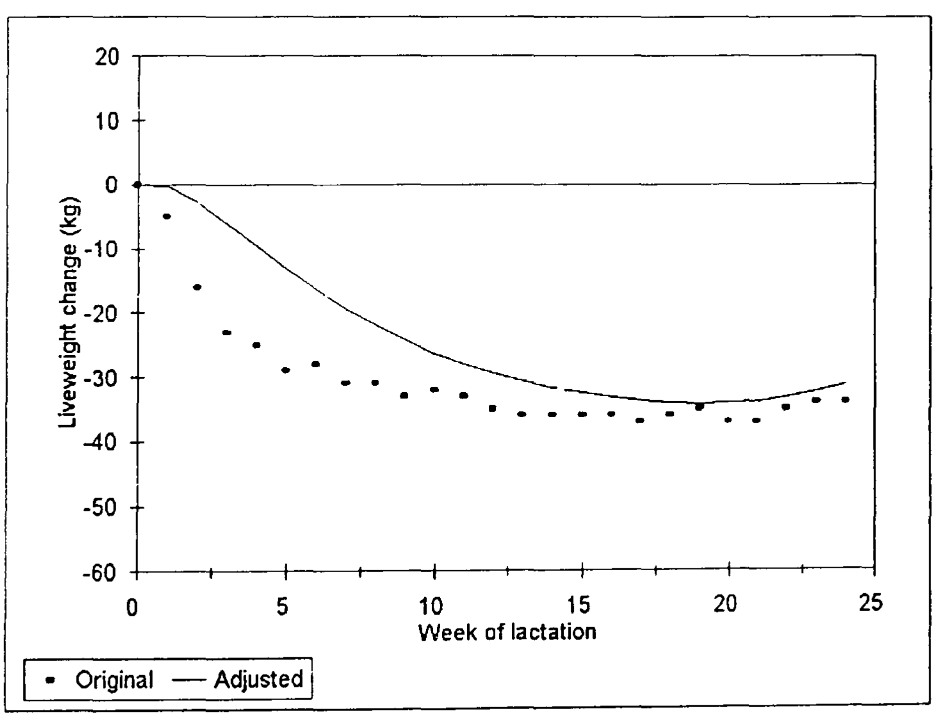



Figure 3.11 - Accumulated liveweight changed originally measured and adjusted (Treatment B)




### 3.3.4. Comparison of the results of the model with the adjusted experimental results

With ($E_y$) estimated by linear regression, and feeding the same amount of energy fed in the experiment, the mathematical model proposed was run with different combinations of $\alpha^+$, $\alpha^-$ and $\gamma$, in order to find the combination with least sum of square errors between predicted and measured milk yields and accumulated liveweight changes.

### 3.3.4.1. Basic lactation curve

The basic lactation curve used for all treatments was based on Wood's model (Wood, 1967) with the parameters fitted to the average milk yield measured in each week during the experiment by Broster et al (1975).

The fitted lactation curve used in the model is shown in equation (3.20):

$$y_n = 0.837 Y n^{0.2332} e^{-0.04n} \tag{3.20}$$

where $y_n$ is the predicted milk yield (kg/day) in week (n)

$Y$ is the predicted peak yield if enough energy is fed to meet the requirements for maintenance and milk production

The model developed by Wood to predict the milk yield was based on data from several experiments. Only milk yield is mentioned in the paper, although those cows certainly had a liveweight change throughout their lactation and it is very likely that they lost weight during their first weeks of lactation.

In order to allow a higher degree of flexibility to the model, a standard liveweight change pattern has been included and the variables $\Delta w^+$ and $\Delta w^-$ are the weight change added to or deducted accordingly.

The standard liveweight change pattern assumed in the model was that followed by MAFF (MAFF, 1984) and described by France et al (1982): a weight loss of 0.5 kg/day during the first ten weeks of lactation.

### 3.3.4.2. Least sum of square errors

Milk yield and liveweight changes were compared to milk yield measured in the experiment and to the adjusted liveweight changes.

For each set ($\alpha^+$,$\alpha^-$,$\gamma$), the sum of square errors (SSE) between predicted and measured milk yield was calculated. Similarly, the sum of square errors between predicted and adjusted accumulated liveweight change was calculated. Treatment M12 was not included in the comparisons due to its anomalous behaviour.

The SSE of milk yield and liveweight changes were much bigger when $\gamma$=0.8, $\gamma$=0.85, $\gamma$=0.9 and $\gamma$=0.95 than when $\gamma$=1.0, with the same pairs of $\alpha^+$ and $\alpha^-$. Hence, only the results with $\gamma$=1.0 were carried on and are shown in Table 3.4. This agrees with the view that once milk yield is lost, it can never be reversed. Note that this is not the same as saying milk yield cannot increase, which it can if energy fed exceeds energy required.

**Table 3.4 - Sum of square errors ($\gamma$=1) of milk yield and accumulated liveweight change**

*Milk yield*

| $\alpha^+$ \ $\alpha^-$ | 0.08 | 0.10 | 0.13 | 0.15 | 0.17 | 0.20 |
|---|---|---|---|---|---|---|
| 0.08 | 111.8 | 117.5 | 130.3 | 140.4 | | |
| 0.10 | 104.8 | 106.3 | 114.7 | 122.4 | 131.3 | 145.3 |
| 0.13 | 109.7 | 105.1 | 106.9 | 111.4 | 117.6 | 128.4 |
| 0.15 | | 110.4 | 108.1 | 110.6 | 115.1 | 123.9 |
| 0.17 | | | | 112.9 | 115.9 | 122.9 |
| 0.20 | | | | | | 125.5 |

*Accumulated liveweight*

| $\alpha^+$ \ $\alpha^-$ | 0.08 | 0.10 | 0.13 | 0.15 | 0.17 | 0.20 |
|---|---|---|---|---|---|---|
| 0.08 | 5740 | 5221 | 5107 | 5316 | | |
| 0.10 | 6348 | 5226 | 4411 | 4250 | 4294 | 4620 |
| 0.13 | 10151 | 8057 | 6097 | 5328 | 4853 | 4531 |
| 0.15 | | 11029 | 8307 | 7132 | 6309 | 5551 |
| 0.17 | | | | 9425 | 8268 | 7088 |
| 0.20 | | | | | | 9967 |

From Table 3.4, we can see that the best pair ($\alpha^+$,$\alpha^-$) for milk yield is not the best pair for accumulated liveweight change. The product of the sum of square errors of both milk yields and accumulated liveweight changes are shown in Table 3.5.

**Table 3.5 - SSEy x SSElw**

| $\alpha^+$ \ $\alpha^-$ | 0.08 | 0.10 | 0.13 | 0.15 | 0.17 | 0.20 |
|---|---|---|---|---|---|---|
| 0.08 | 64.2 | 61.3 | 66.6 | 74.6 | | |
| 0.10 | 66.5 | 55.5 | 50.6 | 52.0 | 56.4 | 67.1 |
| 0.13 | 111.4 | 84.7 | 65.2 | 59.4 | 57.0 | 58.2 |
| 0.15 | | 121.7 | 89.8 | 78.9 | 72.6 | 68.8 |
| 0.17 | | | | 106.4 | 95.8 | 87.1 |
| 0.20 | | | | | | 125.1 |

Milk yield and liveweight changes predicted by the model using the best pair ($\alpha^+$,$\alpha^-$), from Table 3.5, are shown in Appendix III.

Predicted values from the mathematical model are plotted together with actual milk yield and adjusted liveweight change in Figure 3.12 to Figure 3.17; Treatments MG, M16, M12, M8, M4 and B, respectively.

Figure 3.12 - Predicted and actual milk yield and liveweight changes (Treatment MG)

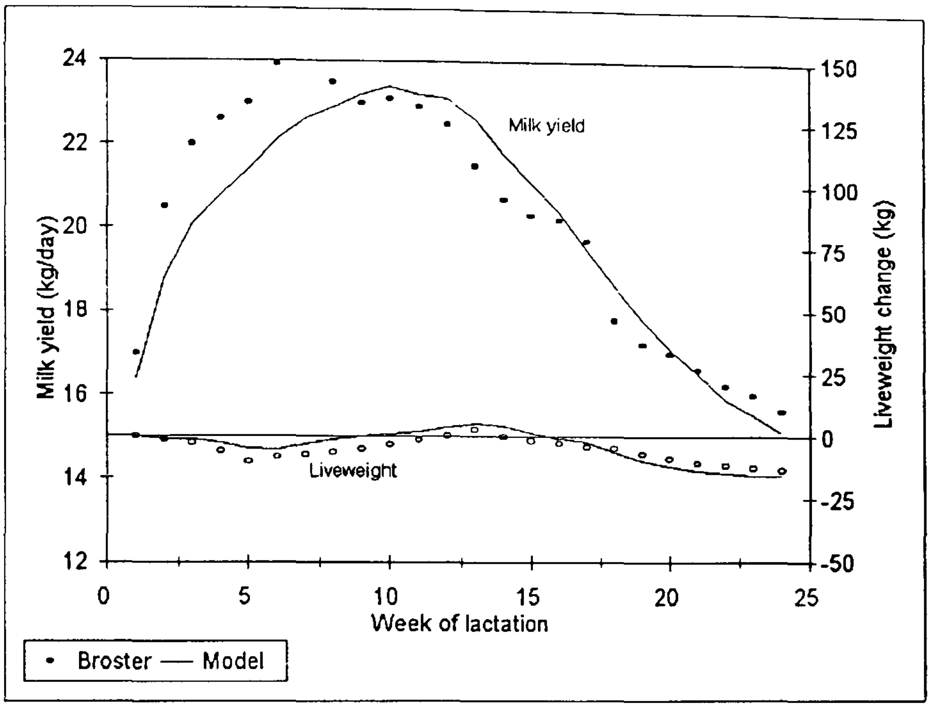



Figure 3.13 - Predicted and actual milk yield and liveweight changes (Treatment M16)

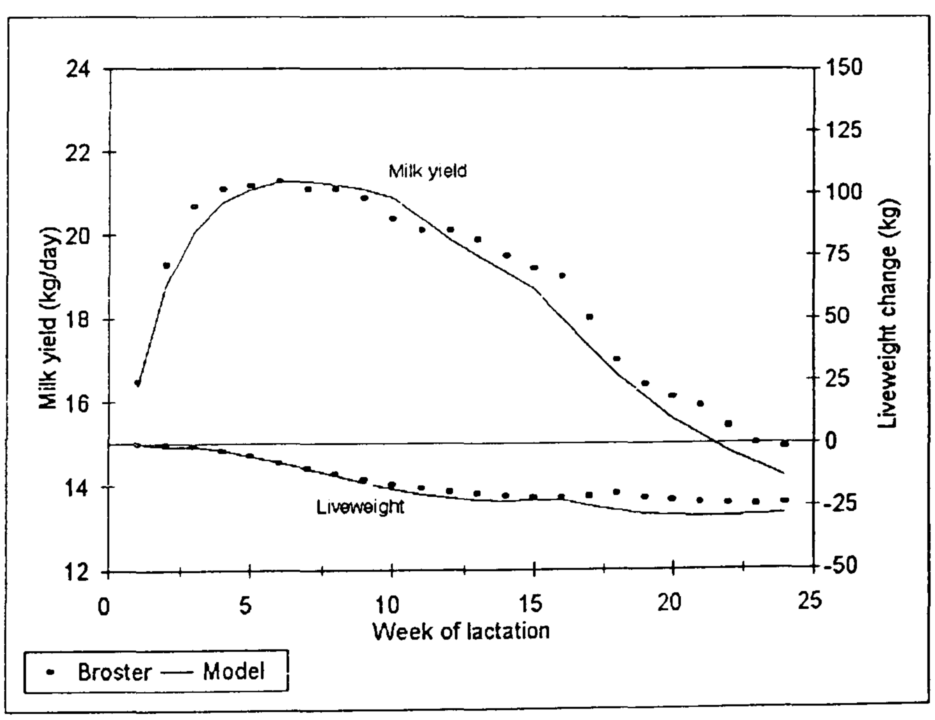


Figure 3.14 - Predicted and actual milk yield and liveweight changes (Treatment M12)

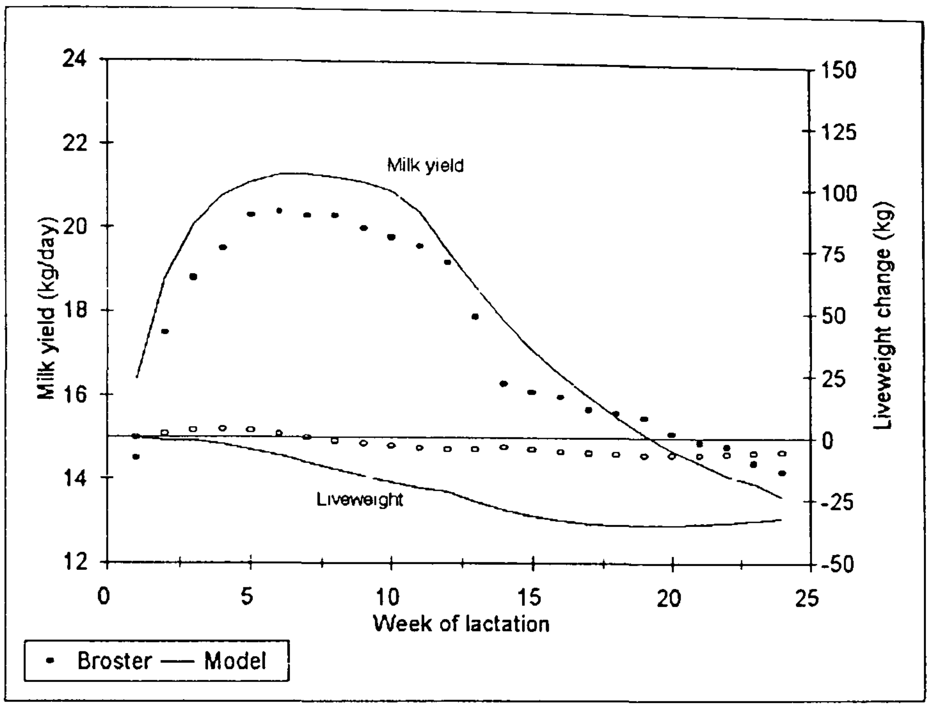



Figure 3.15 - Predicted and actual milk yield and liveweight changes (Treatment M8)

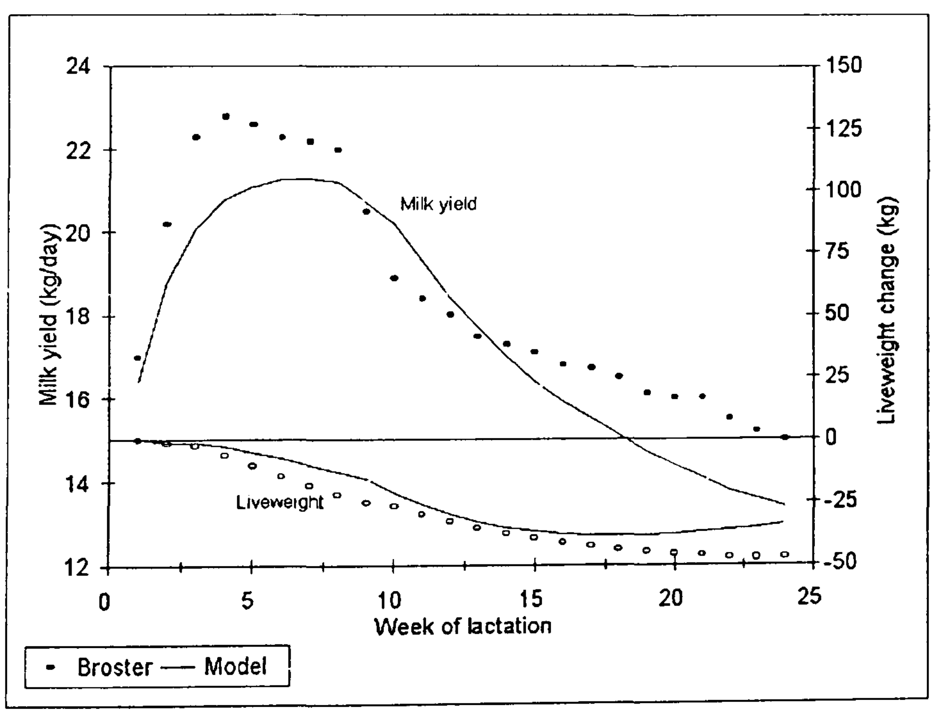


Figure 3.16 - Predicted and actual milk yield and liveweight changes (Treatment M4)

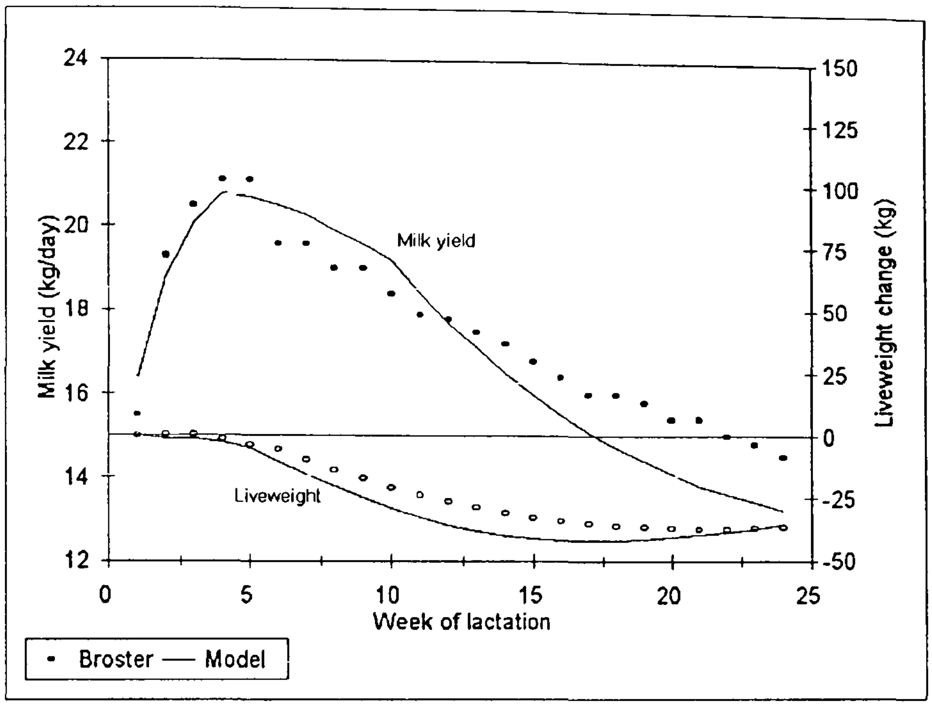
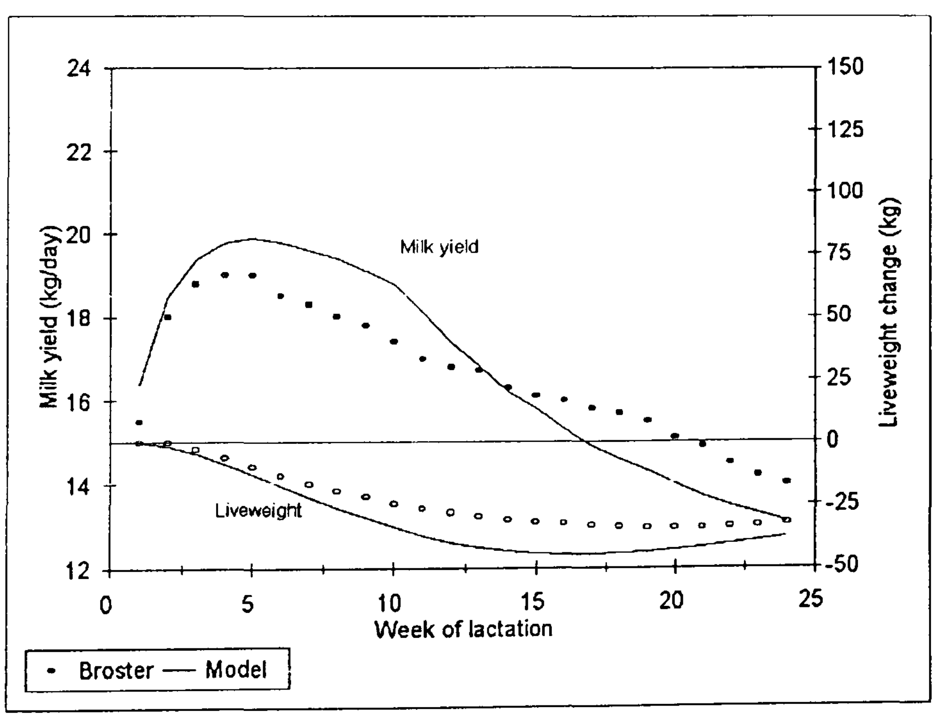



Figure 3.17 - Predicted and actual milk yield and liveweight changes (Treatment B)




When the results from Broster et al (1975) are compared with the results from the mathematical model to describe the effects of feeding levels on liveweight changes and on milk production, it can be seen that, allowing for experimental error, the behaviour of the model is acceptable. Using only the level of feeding and the partition parameters $\alpha^+$ and $\alpha^-$ for under and over feeding, the model is well able to reproduce milk yield curves varying from a peak of 24 kg/day ending at 16 kg/day in week 24 to a peak of 20 kg/day ending at 13 kg/day in week 24.

Milk yields and liveweight changes predicted by the model are close enough to those originally measured by Broster et al (1975), except for Treatment M12, which was anomalous, anyway.

The mathematical model described in this chapter is part of the Linear Programming (LP) model, which optimizes the whole dairy farm system. It gives the LP a higher flexibility to find the optimum strategy for feeding cows, optimizing simultaneously the strategy for weight loss and weight gain rather than following a pre-determined weight change pattern.

# Chapter Four

## 4.The Linear Programming Model

A linear programming (LP) model has been developed to represent a dairy farm system. It is a multi-period model that represents the several stages of the lactation and the grass growth season throughout the year.

One of the main features of the model is how feeding level throughout lactation and the associated milk yield, weight loss and weight gain have been approached. These are decision variables over which some control is possible, as this gives a much greater flexibility to the model. The optimal feeding strategy and optimal land use (and schedule) are determined simultaneously with the optimum weight change pattern.

The LP can be divided into the following sections:

- **land use and schedule of activities (section 4.3.1)**
  Decision variables concerning land use (maize and cash crop areas, silage-making and grazing areas) which follow the constraints concerning farm size, sequence of land use, time available to make silage as a function of technical coefficients such as work rate of the system being used to make silage.
- **nutritional requirements and feeding components (section 4.3.2)**
  Feeding components (grass silage, maize silage, concentrates and grazing areas) which provide the nutrients required by dairy cows for their maintenance, pregnancy, weight change and milk production in each stage of their lactation. Constraints limiting the amount to be eaten (maximum DM intake and maximum proportion of concentrates and maize silage in the ration) are considered. The ensiling process and the inter-relationship between grass silage into silo and grass silage fed from the silo are also taken into account.
- **liveweight (LW) change (section 4.3.3)**
  Decision variables concerning LW gain or loss are determined using a set of constraints which limit weight changes in each period and the maximum weight loss allowable in cows.
- **milk production (section 4.3.4)**
  Milk yield is limited by quota and is definitely affected by feeding level so this section has a strong relationship with the previous section (LW change).
- **net margin (section 4.3.5)**
  This is the objective function of the LP model therefore expenditure and income from the components of the dairy farm system have to be considered.

## 4.1. Assumptions

We make the following realistic assumptions in the model:

- Grass growth season lasts 26 weeks.
- Grass can be grazed off up to the 5th week of the season; 2-, 3- and 4- week regrowth periods are allowed for grass to be grazed; otherwise grass quality is too low and could never be considered for grazing.
- First growth grass must be harvested between the 5th and the 7th week of the season; regrowth for 5, 6 or 7 weeks is allowed; it is assumed farmers would never consider cutting before due to low yield and after due to low quality.
- Grass from harvested areas has a delay of one week before it starts to grow again, compared to the growth rate assumed for grazed grass.
- Total area must be used; grass area must be either grazed or harvested.
- There is one silo for maize silage and two for grass silage: one for silage made with first-cut grass (higher quality) and one for silage made with grass from second and third cuts (lower quality).
- Grazing efficiency is assumed to be 60% (Parsons, 1993); this means that there is a loss of 40% of the DM yield of the grass because of the damage to the grass caused by the herd.
- Cows are supposed to stop losing weight once they become pregnant; hence, the model includes variables representing weight loss only during the first 16 weeks of lactation.
- There is a maximum daily weight change allowed; this is assumed to be constant.
- A "basic weight change pattern" based on MAFF's recommendation (MAFF, 1984) is assumed. The decision variables concerning weight changes represent weight changes in addition to the "basic weight change pattern" cows are assumed to have.
- It is assumed that cows have an expected milk yield ("basic milk yield") throughout the lactation; if they are fed to have the "basic weight change", it is assumed that they produce the "basic milk yield". Any weight gain or loss will affect the actual milk production by increasing or decreasing the capability of cows to produce milk compared to the "basic lactation curve".
- The year is divided in periods; $S_1$ has thirteen 4-week periods and indicates the calving periods; $S_2$ has thirty-three periods and is divided in 1-week periods during the grass growth season and in 4-week periods otherwise (see section "Periods of the year" in Chapter 4 for details of the periods and their dates).

## 4.2. Notation for the LP model

### Variables

$g_{ijk}$ : Area of (k) week old grass grazed in period (j) by cows calving in period (i) (ha). When grazed off, k=0.

$h_{jk}$ : Area of grass harvested in period (j) after (k) weeks of regrowth (ha). When first cut, k=0.

$a_z$ : Maize crop area (ha).

$a_c$: Cash crop area (ha).

$n_i$ : Number of cows calving in period (i).

$w_{ij}^{+}$ : Weight gain in period (j) by cows calving in period (i) (kg).

$w_{ij}^{-}$ : Weight loss in period (j) by cows calving in period (i) (kg).

$s_{ijq}$ : Grass silage quality (q), eaten in period (j) by cows calving in period (i) (tDM).

$z_{ij}$ : Maize silage eaten in period (j) by cows calving in period (i) (tDM).

$c_{ijq}$ : Concentrates type (q), eaten in period (j) by cows calving in period (i) (tDM).

$q_{DM}$ : Total DM of grass silage available from silo after losses (tDM).

$q_{ME}$ : Total ME available of grass silage from silo after losses (MJ).

$y_{ij}$ : Actual milk yield in period (j) of cows calving in period (i) (kg milk).

$h_{mach}$ : Number of sets of machinery (i.e., forage harvester, tractors and trailers) to make grass silage (including labour).

### Parameters

A : Total area of farm (ha).

$R_{jk}$ : Work rate when making silage in period (j) with grass (k) weeks of regrowth (ha/h).

T : Time available for making silage (h).

$DMI_{ij}$ : DM intake capacity of any forage during period (j), for cows calving in period (i) (tDM).

$ME_{ij}$ : Metabolizable energy (ME) required for maintenance, pregnancy and expected milk production in period (j) by cows calving in period (i) (MJ).

$Y_{ij}$ : Expected milk yield in period (j) of cows calving in period (i) (kg)

$DMg_{jk}$ : DM yield of field grazed in period (j) after (k) weeks of regrowth (tDM/ha).

$MEg_{jk}$ : ME content of grass being grazed in period (j) after (k) weeks of regrowth (MJ/kgDM).

$\eta_g$ : Grazing efficiency. Factor that relates the amount of grass actually grazed by cows to the total amount of grass available.

$L_c$ : Maximum proportion of concentrates in the ration.

$L_z$ : maximum proportion of maize silage in the ration.

$DMh_{jk}$ : DM yield of field grass harvested in period (j) after (k) weeks of regrowth (tDM/ha).

$MEh_{jk}$ : ME content of silage (after losses) made with grass from harvested in period (j) after (k) weeks of regrowth (MJ/kgDM).

DMz : DM yield of maize crop (tDM/ha).

$DMz_{loss}$ : dry matter loss of maize silage (both field and in-silo losses).

MEz : ME content of maize silage (MJ/kgDM).

$DMs_{loss}$ : dry matter loss of grass silage (both field and in-silo losses).

$MEs_q$ : ME content of grass silage quality (q) (MJ/kgDM).

$MEc_q$ : ME content of concentrate quality (q) (MJ/kgDM).

$S_{ef}$ : substitution effect factor due to concentrate intake.

f : Factor relating milk yield to DM intake.

$\delta^+/\delta^-$ : ME requirement increase or decrease due to LW gain or loss (MJ/kg LW change).

$\lambda^+/\lambda^-$ : Effect of LW gain or LW loss on milk yield (kg milk/kg LW change).

$T_{loss}$ : Maximum total weight loss that cows are allowed to lose (kg).

$W_f$ : Final LW of the "average cow" of the herd (kg).

$W_0$ : Initial LW of the "average cow" of the herd (kg).

$L_j$ : Maximum weight change (loss or gain) in period (j) (kg).

Q : Annual milk quota (kg milk).

$P_l$ : Opportunity cost of land for cash crop (£ / ha)

$P_j$ : Milk price in period (j) (£ / kg milk)

$P_g$ : Annual cost for grazing areas (£ / ha)

$P_h$ : Annual cost for grass silage areas (£ / ha)

$P_z$ : Annual cost for maize crop areas (£ / ha)

$Pc_K$ : Price of conentrates (£ / tDM)

$D_c$ : Annual depreciation of cows (£ / cow / year)

$D_{mach}$ : Annual depreciation of specialist machinery (£ / year)

$S_1$ : Group of thirteen 4-week periods into which the year was divided (see Section 6.1 for details)

$S_2$ : Group of thirty-three periods with different lengths during grass growth season (see Section 6.1 for details)

## 4.3. Description of the model (constraints and variables)

### 4.3.1. Land use and management

*a. Area constraints (farm size)*

The farm area can be allocated to grass for grazing and harvesting, or cash crop and maize can be grown on the farm so they may compete with grass for the same land. It is assumed in the model that both cash crops and maize are harvested by contractors if they are grown.

$$\sum_{i \in S_1} \sum_{j=1}^{5} g_{ij0} + \sum_{j=5}^{7} h_{j0} + a_z + a_c = A \tag{4.1}$$

Total area available must be allocated to grass (either for grazing or silage), maize or cash crop.

*b. Sequence constraints*

Grass growth rate varies throughout the season and a higher rate occurs in May and June, when the excess not grazed is made into silage for winter feeding. Grass must be cut to maintain its quality in future regrowths. The optimal sequence and time for grazing and harvesting are determined based on the yield and on the quality of the grass, which varies over the season.

$$g_{ij0} + \sum_{k=2}^{4} g_{ijk} + h_{j-1,0} + \sum_{k=5}^{7} h_{j-1,k} = \sum_{k=2}^{4} g_{i,j+k,k} + \sum_{k=5}^{7} h_{j+k,k} \tag{4.2}$$

$$\forall i \in S_1,\ j = 1 \cdots 26,\ j > k,\ j + k \le 26,\ \exists g_{j,0},\ \exists h_{j,0}$$

Total grazed area in a given week becomes available to be grazed or harvested (k) weeks later. Grass starts to regrow with a delay of one week in fields where it has been harvested. So, it becomes available to be grazed or harvested again k+1 weeks later.

*c. Forage harvesting system constraints*

Silage must be made within the maximum time allowed (T) to make silage. Equation (4.3) shows that silage made with first cut grass must be made within the maximum time allowed. Similarly, equation (4.4) shows the limitation for other cuts.

$$\sum_{j} h_{j0} \, R_{j0} \le T \qquad j \in S_2 \,/\, \exists h_{j0} \tag{4.3}$$

$$\sum_{j} h_{jk} \, R_{jk} \le T \qquad j \in S_2 \,/\, \exists h_{jk} \tag{4.4}$$

Throughput capacity of harvesters restricts the areas with high yields which can be harvested. Limitation in areas with low yield is due to harvester's forward speed. These limitations are considered when work rate is calculated (Section 5.5.2.3).

### 4.3.2. Relationship between energy requirements, appetite and feeding strategy

Nutritional requirements and maximum dry matter (DM) intake capacity of cows vary throughout the lactation. Cows are fed with a combination of available components and are allowed to graze (during grass growth season) in order to get the nutrients they need.

One can buy in any feed from straw and maize/silage to grains/sugarbeet pulp. In this LP model, only the major items generally available are being considered. Grazing is normally a cheap source of nutrients but grass is not available all the time. Grass can be conserved and fed as silage during winter. Alternatively, maize can also be cropped and conserved at the farm and fed later as silage. Concentrates are normally the most expensive component which are used to make up a suitable ration to allow the cow to produce the targeted milk yield. The latter are a blend of several feeds with high energy and protein levels.

In this section of the LP, the optimal amount of each component to be fed is evaluated. Grazing areas are determined directly (e.g., the decision variables concerning grazed areas represent the actual area grazed by cows). Grass silage and maize silage have an intermediary step: the ensiling process during which time there are losses. The model links the decision variables representing grass and maize areas with the decision variables representing grass and maize silage (this is achieved by using transfer constraints).

There are two additional constraints in the model to limit the proportion of concentrates and maize silage in the ration.

*d. Constraints limiting dry matter intake (appetite)*

Total dry matter intake from grazing areas, grass silage, maize silage and concentrates is limited by the maximum DM intake capacity of cows in each period. DM intake capacity of cows varies according to the lactation stage. Liveweight changes also affect the DM intake (weight gain increases the DM intake and weight loss reduces the DM) in a cumulative way.

$$\sum_k DMg_{jk} \cdot \eta_g \cdot g_{ijk} + \sum_q s_{ijq} + z_{ij} + \sum_q S_{ef} \cdot c_{ijq} \le DMI_{ij} \cdot n_i + f \sum_{n=1}^{j} w_{in}^{+} - f \sum_{n=1}^{j} w_{in}^{-}$$

$i \in S_1,\ j \in S_2,\ k = 0, 2, 3, 4$ **(4.5)**

*e. Energy balance constraints*

Total energy fed (from all sources of energy) must be equal to the total energy required by cows in each period. Total energy required by cows includes energy for maintenance, pregnancy and milk production (dependent on the lactation stage), and energy required for weight loss or released by weight gain.

$$\sum_k MEg_{jk} \cdot DMg_{jk} \cdot \eta_g \cdot g_{ijk} + \sum_q MEs_q s_{ijq} + MEz \cdot z_{ij} + \sum_q MEc_q \cdot c_{ijq} =$$

$$\mathrm{ME}_{ij} \cdot n_i + \delta^{+} \sum_{n=1}^{j} w_{in}^{+} - \delta^{-} \sum_{n=1}^{j} w_{in}^{-}$$

$i \in S_1,\ j \in S_2,\ k = 0, 2, 3, 4$ **(4.6)**

Energy provided by grass silage ($MEs_q$) depends on the time that grass is ensiled and this is a variable. Since there is a product of two decision variables ($MEs_q.s_{ijq}$), equation (4.6) is not linear and a recursive approach in which the value of $MEs_q$ is successively estimated is adopted in the model to deal with the non-linearity (see equations (4.9) to equations (4.11) and section 6.2 for details of the recursion approach).

*f. Constraints limiting the proportion of concentrates in the ration*

There is a maximum proportion of concentrates allowed in the ration fed each period.

$$\frac{\sum_q c_{ijq}}{\sum_k DMg_{jk} \cdot \eta_g \cdot g_{ijk} + \sum_q s_{ijq} + z_{ij} + \sum_q c_{ijq}} \le L_c$$

$i \in S_1,\ j \in S_2,\ k = 0, 2, 3, 4$ **(4.7)** *g. Constraints limiting the proportion of maize silage in the ration*

There is a maximum proportion of maize silage allowed in the ration fed each period.

$$\frac{z_{ij}}{\sum_k DMg_{jk} \cdot \eta_g \cdot g_{ijk} + \sum_q s_{ijq} + z_{ij} + \sum_q c_{ijq}} \le L_z \qquad i \in S_1,\ j \in S_2,\ k = 0, 2, 3, 4 \qquad \textbf{(4.8)}$$

*h. Grass silage bank constraints (silo)*

Equation (4.9) represents the total amount of DM ensiled that will be available as silage (after losses). Equation (4.10) limits the total amount of grass silage fed to that available in the silo. Equation (4.11) represents the total ME ensiled that will be available as silage (after losses).

$$\sum_k (1 - DMs_{loss}).DMh_{jk}.h_{jk} = q_{DM} \quad j \in S_2 / \exists h_{jk},\ k = 0,2,3,4 \qquad \textbf{(4.9)}$$

$$\sum_{i \in S_1} \sum_{j \in S_2} \sum_q s_{ijq} \le q_{DM} \qquad \textbf{(4.10)}$$

$$\sum_k MEh_{jk}.(1 - DMs_{loss}).DMh_{jk}.h_{jk} = q_{ME} \quad j \in S_2 / \exists h_{jk},\ k = 0,2,3,4 \qquad \textbf{(4.11)}$$

$q_{DM}$ could be eliminated by merging equations (4.9) and (4.10), but the value of $q_{DM}$ is required by the recursive approach to determine the grass silage quality. The grass silage quality ($MEs_q$) is recursively calculated using $q_{DM}$ and $q_{ME}$ ($MEs_q = q_{ME}/q_{DM}$) from equations (4.9) and (4.11) (see Section 6.2. for details of the recursion approach to determine the grass silage quality).

*i. Maize silage bank constraints (silo)*

The total amount of maize silage fed in each period is limited by the amount of maize cropped and ensiled.

$$\sum_{i \in S_1} \sum_{j \in S_2} z_{ij} \le (1 - DMz_{loss}).DMz.a_z \qquad \textbf{(4.12)}$$

## 4.3.3. Liveweight change

In terms of the dairy cow energy must be conserved. Milk production only varies slowly in response to under or overfeeding, thus on a daily basis cows lose weight when they are underfed energy and gain weight when are overfed energy. There is a liveweight (LW) change pattern followed by most models, which is the LW change if cows are fed according to the energy requirements estimated by MAFF (1984). Cows are usually fed to maintain that weight change pattern throughout their lactation.

When cows are overfed, part of the surplus energy is allocated to milk production and part to weight gain. When they are underfed part of their body reserves is converted into energy to produce milk, provoking a weight loss during the process (see Chapter 3).

An excessive weight loss may cause severe consequences to cows (MAFF, 1984; NRC, 1988). Hence, the total LW loss is limited to a maximum loss and a set of constraints of the model represents this limitation.

By the time mature cows calve again they should have gained the weight they lost during lactation. First and second lactation heifers and third lactation cows usually gain more weight than they lose because they are still growing. The total weight gain above or equal to the total weight loss brings the cow to better condition to calve. Initial and targeted final LW of cows will vary according to their age and parity. The herd structure (i.e., the percentage of cows of the herd in each parity) will determine the initial and targeted final LW of the "average cow" of the herd. They will need to gain weight to achieve that targeted weight by the time they calve. There is another set of constraints forcing cows to gain at least the weight they lost plus the weight necessary to achieve the targeted final LW. There is also a set of constraints that limit the weight change of cows in each period.

*j. Constraints limiting total weight loss*

The accumulated weight loss is limited by a maximum total weight loss.

$$\sum_{j \in S_2} w_{ij}^- \leq T_{loss} \qquad i \in S_1 \qquad \textbf{(4.13)}$$

*k. Minimum total weight gain constraints*

Cows must gain weight to achieve at least the targeted final liveweight.

$$\sum_{j \in S_2} w_{ij}^+ \geq \sum_{j \in S_2} w_{ij}^- + W_f - W_0 \qquad i \in S_1 \qquad \textbf{(4.14)}$$

*l. Constraints limiting weight change in each period*

Weight gain or loss in each period is limited. Notice that cows cannot gain and lose weight simultaneously; then, constraints represented by equation (4.15) limit either weight gain or weight loss in each period.

$$w_{ij}^{+} + w_{ij}^{-} \le L_j \qquad i \in S_1,\ j \in S_2 \tag{4.15}$$

## 4.3.4.Milk production

Milk yield throughout the lactation is determined by the cows' potential (or expected) milk yield and by the effect of the level of feeding, which is directly linked to LW changes. This effect is cumulative and a set of constraints links this effect, connecting each period to the subsequent periods.

*m. Milk production constraints*

Actual milk yield is the expected milk yield plus the accumulated effect of liveweight change. When cows gain weight (surplus energy), there is a positive effect on the milk production. The opposite occurs when cows lose weight (deficit energy), but with different magnitude.

$$y_{ij} = Y_{ij}.n_i + \lambda^{+}\sum_{n=1}^{j} w_{in}^{+} - \lambda^{-}\sum_{n=1}^{j} w_{in}^{-} \qquad i \in S_1,\ j \in S_2 \tag{4.16}$$

*n. Milk quota constraint*

Total milk production is limited by the farm milk quota.

$$\sum_{i \in S_1}\sum_{j \in S_2} y_{ij} \le Q \tag{4.17}$$

## 4.3.5.Net margin: income less costs

The objective function of the model is to maximize net margin (income less costs). Cash crops and milk are the two components that can be sold; they are the only sources of income in the model. Conversely, there are many components that reduce the net margin. Some of these costs are variable costs such as costs to grow grass (e.g., seed, fertilizer, sprays, etc.), costs to produce silage (e.g., harvesting costs), cost of concentrates and labour. Some of these costs are fixed costs such as machinery depreciation. Machinery was categorised as specialist (used only for forage conservation operations) or multi-purpose (shared with other operations on the farm) as in Corrall et al (1982) and McGechan (1990c). For the specialist machines, the whole annual cost is considered while for the shared machines, the hourly cost to the forage operations has been calculated from the total annual cost, assuming a total annual usage of 500 h for tractors and 200 h for other items. Cow depreciation is also taken into account and is calculated based on the replacement rate of theherd, cost of 20-month old heifers, cull cow price and sundry costs such as artificial insemination, veterinary charges, bedding, etc.

*o. Objective function*

$$\begin{aligned} Max \quad & \sum_{i \in S_1} \sum_{j \in S_2} P_j . y_{ij} - \sum_{i \in S_1} \sum_{j \in S_2} P_g . g_{ij0} - \sum_{j \in S_2} P_h . h_{j0} - P_z . a_z \\ & - \sum_{i} \sum_{j} \sum_{q} Pc_q . c_{ijq} - \sum_{i \in S_1} D_n . n_i - D_{mach} . h_{mach} + P_l . a_c \end{aligned} \qquad \textbf{(4.18)}$$

# Chapter Five

## 5. Data and mathematical models to predict data

The linear programming (LP) that represents a dairy farm system needs a large amount of data that can be divided in two groups:

- economic data concerning prices and costs
- technical data related to biology and machinery.

Economic data can be easily found in commercial publications (e.g., Farmers Weekly, Dairy Farm, Farm Management Pocketbook, by John Nix, and ABC - Agricultural Budget Costs) normally published once a year. These sources provide information such as compositional milk prices and their seasonal variations, fertilizer prices, machinery costs, etc. In some cases there are minor differences from one source to another, but generally, most figures are similar.

However, problems arise when technical data are gathered from literature. Numerous formulae and mathematical models have been developed to estimate or predict data and significant differences can be found, usually due to different field observations, adjustments to local conditions or conditions during experimentation.

The LP model uses a combination of data from different sources which has been carefully selected in order to maintain an equilibrium with the system and to avoid incompatibility between data or even unfeasibility of the LP.

In this chapter, a review is presented of the sources for the technical data used by the LP model. The majority of data is predicted or estimated by mathematical models that provide technical coefficients to the LP matrix and these mathematical models form the main components of the dairy farm system.

The main components of the LP model are:

- cow-dependent
  - milk production
  - appetite
  - energy requirements
- feed-dependent
  - grass crop (seasonal growth and digestibility)
  - silage
    - grass silage
      - conservation (nutritional values, and field and in-silo losses)
      - field operations (machinery, silage-making system and workrate)
    - maize silage
  - concentrates

## 5.1. Milk production

Milk production increases quickly from calving to a peak, a few weeks later, when gradual decline starts and lasts until about ten months after calving. The rate the yield increases during the first weeks of lactation varies mainly with the breed and the age of the cow (Hulme et al., 1986; Olney and Falconer, 1985; Wood, 1969; France et al, 1982). The peak and the rate at which the yield decreases can also be linked to the breed.

The milk production adopted in the LP model is represented by the lactation curve

$$y_n = A\,n^B e^{-C n} \tag{5.1}$$

where (yn) is the average daily milk yield in week (n), (A) is the scale parameter and (B) and (C) are the shape parameters of the curve (Wood, 1967).

The turning point of the continuous equivalent of equation (5.1) can be easily calculated and occurs when $n = \left(B/C\right)$. Hence, peak milk yield ($Y$) is

$$Y = A\left(B/C\right)^B e^{-B} \tag{5.2}$$

The LP model uses mean values for (B) and (C) obtained by Wood (1969) in an analysis with more than 800 Friesian lactations for first, second, third and subsequent lactations. From equation (5.2), we can calculate

$$A = \left[\left(C/B\right)^B e^B\right] Y \tag{5.3}$$

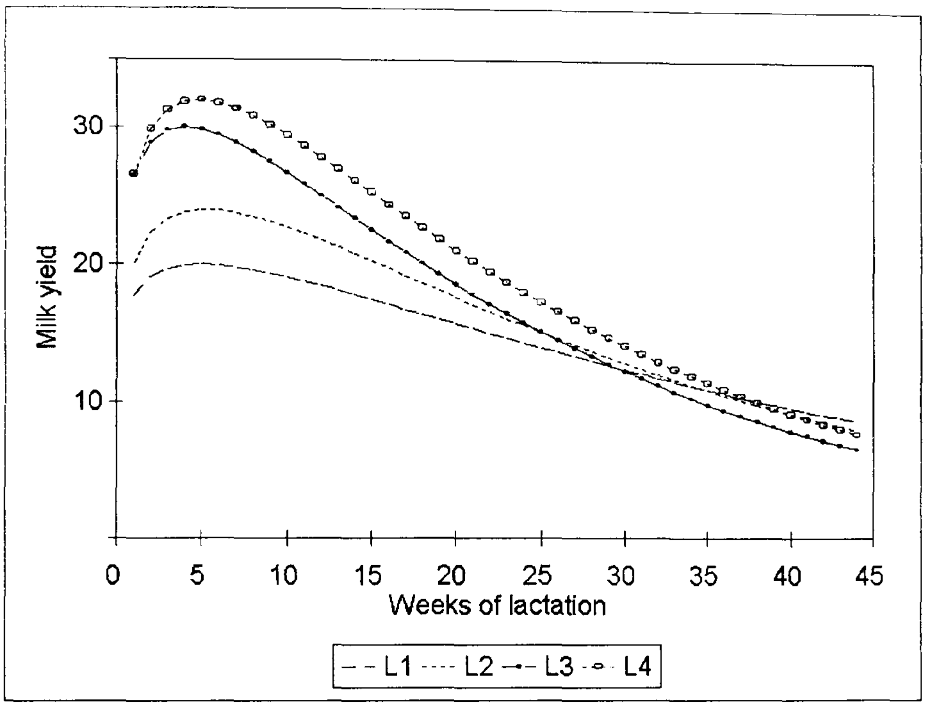
We assume that the lactation lasts 305 days (approximately 44 weeks). The total milk yield ($y_{tot}$) over the lactation can be calculated from equations (5.1) and (5.3), by calculating the sum of the milk yield ( $y_{tot} = \sum_{n=1}^{44} An^B e^{-Cn}$ ). For example, when (B=0.2) and (C=0.4), the peak occurs at the fifth week of lactation and the total yield is approximately 208 times the peak yield ($y_{tot} \approx 208$ Y).

Table 5.1 shows the mean values of A, B and C for each lactation and Figure 5.1 shows the lactation curves for the corresponding parameters. (Wood, 1969).

**Table 5.1 - Lactation curve parameters and estimated total milk yield**

| Lactation | A | B | C | Total yield |
|---|---|---|---|---|
| 1 | 0.9126 Y | 0.15 | 0.03 | ≈ 227 Y |
| 2 | 0.8709 Y | 0.21 | 0.04 | ≈ 210 Y |
| 3 | 0.9257 Y | 0.20 | 0.05 | ≈ 182 Y |
| ≥ 4 | 0.8724 Y | 0.24 | 0.05 | ≈ 190 Y |

**Figure 5.1 - Lactation curves for each lactation (kg milk/day)**




Wood (1969) associates the effect of parity on the shape of the curve, particularly its uniformity, to a combination of service intervals with higher potential yields. The older the cow the higher the yield level at which she starts her lactation. The pregnancy, however, occurs at about the same stage of lactation for all levels of production, so the rate of decline accelerates in older cows.

Experiments on the food utilization by dairy cows (Broster, 1975; Poole, 1986) show that the feed level in early lactation strongly affects the future milk yield. When cows received lower-energy diets in early lactation, the weight lost was difficult to regain and their milk yields declined during later stages of lactation. In contrast, cows that received higher-energy diets in early lactation lost less weight and produced more milk over the lactation. Furthermore, changes in the transformation of nutrients were evident, in mid lactation, to milk rather than to body reserves. These experiments show that the energy diet has a long term effect on the milk production. This is particularly true of the diet offered during early lactation.

## 5.2. Appetite (Dry Matter Intake)

Cows are mainly fed forage and concentrates. The amount of dry matter (DM) eaten by cows is known to be greatly influenced by their liveweight, level of milk production and stage of lactation (MAFF, 1984; NRC, 1988; Brown et al., 1977; Hulme et al., 1986). Type and quality of food also have an important role in the appetite of cows, especially forage intake. If cows are fed low energy food, they may lose weight unless concentrates are provided. Concentrates are much more expensive than forage and high levels of concentrates in the ration are inadequate for normal fermentation in the rumen so this reduces milk fat production drastically (NRC, 1988). Therefore, it is necessary to find an adequate balance between forage and concentrates throughout the lactation.

Appetite is known to be reduced during early lactation. Sometimes it is impossible to provide the energy required within the appetite limits, especially for high-yielding dairy cows in their first weeks of lactation. As shown by Broster (1975), it is difficult to recover from a reduction in milk yield at early lactation. The long-term effect of feeding dairy cows inadequately during early lactation means that the prediction of dry matter intake (DMI) has an important role in the whole system.

An expression for predicting dry matter intake presented in ARC (1980) is

$$DMI = \left[\ 0.135\ LW^{0.75} + 0.2\left(Y - Y_{5000}\right)\ \right] f_{xt} \tag{5.4}$$

where DMI : dry matter intake (kgDM/day)

LW : liveweight (kg) and

$Y$ : daily milk yield (kg/day)

$Y_{5000}$ : average milk yield (kg/day) at that week, when total lactation is 5000 kg; $y_{5000} = a\ n^{b}\ e^{-cn}$ (Wood, 1967) with a=21.4, b=0.2 and c=0.04

$f_{xt}$ : adjustment for the variation in intake during lactation (Table 5.2).

**Table 5.2 - Dry Matter Intake adjustment during lactation**

| t, month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f_{xt}$ | 0.81 | 0.98 | 1.07 | 1.08 | 1.09 | 1.08 | 1.01 | 0.99 | 0.97 | 0.97 |

Parsons (1992b) suggests some extended values of fxt, in order to prevent a large step from 0.97 to 0.81, before calving. The values are evened out by interpolating the tabulated values, as shown in Table 5.3.

**Table 5.3 - Dry Matter Intake adjustment during the dry period**

| t, month | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|
| $f_{xt}$ | 0.93 | 0.93 | 0.93 | 0.93 | 0.85 |

During the dry period, time is calculated backwards, with the month just before calving corresponding to month 15 of the table. Therefore, fxt is always 0.85 immediately before calving, regardless of the calving interval.

Many experiments suggest that body weight and milk yield are the most important determinants of total intake of dry matter (MAFF, 1984; NRC, 1988; Brown et al., 1977; Hulme et al., 1986). Equation (5.4) takes account of the stage of lactation ($f_{xt}$ is an adjustment related to the month of lactation) and the milk yield level. Neal et al. (1984) and Parsons (1992b), however, found that dry matter intakes for high-yielding cows predicted by that equation may not be suitable.

Hulme et al. (1986) describe a model to calculate dry matter intake, which takes account of the body weight, milk yield, stage of lactation, relative edibility of food and substitution effect. The authors claim their calculations allow a more accurate prediction of dry matter intake. The calculations are used in a model to formulate rations but they are too complex to be included in an LP model that optimizes the whole dairy farm system.

A simpler model to predict dry matter intake (DMI) is presented in MAFF (1984) and has been found generally useful during mid and late lactation. Neal et al. (1984) state that MAFF's model to predict dry matter intake performs reasonably well and could be used when only liveweight and milk yield are available. It is also suggested that a slight adjustment for the beginning of the lactation should be used in order to give acceptable predictions. This equation will be used to calculate the dry matter intake for the LP model.

$$DMI = 0.025\,LW + 0.1\,Y \qquad \textbf{(5.5)}$$

where $DMI$ : dry matter intake (kg DM/day)
$LW$ : liveweight (kg)
$Y$ : milk yield (kg/day).

In order to take into account the reduction of the appetite in early lactation, MAFF (1984) recommends a reduction of 2-3 kg/day below the values given by equation (5.5) during the first ten weeks of lactation, to give a better prediction during that period. The reductions used for the LP model were changed linearly and are shown in Table 5.4.

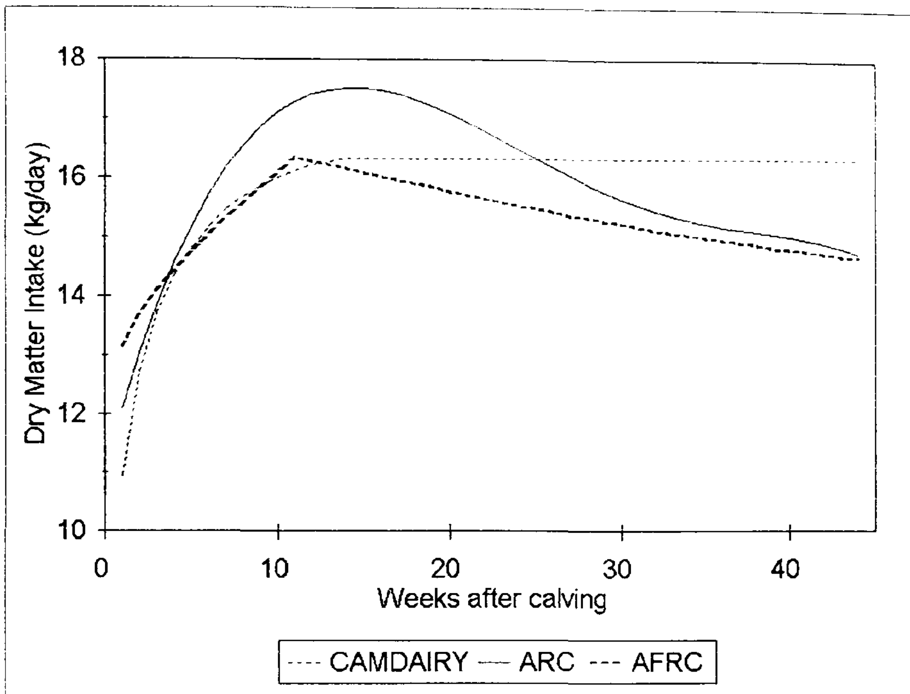
**Table 5.4 - Reductions of appetite during early lactation (kg/day)**

| Week | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Reduction | -3.0 | -2.7 | -2.4 | -2.1 | -1.8 | -1.5 | -1.2 | -0.9 | -0.6 | -0.3 |

It is assumed in this LP model that DMI will vary only with milk yield and stage of lactation, since weight change is fat and weight changes seem unlikely to effect capacity for intake. Therefore, in this model the weight to calculate the DMI is the cows' initial liveweight.

Figure 5.2 illustrates the dry matter intake during lactation predicted by equations (5.4) and (5.5), and by the model described in CAMDAIRY (Hulme et al, 1986) for a cow with 550 kg, milk peak yield of 28 kg/day and (B=0.2) and (C=0.04).

**Figure 5.2 - Dry Matter Intake predicted by CAMDAIRY, ARC and AFRC equations**




## 5.3. Energy requirements

Cows require energy for the maintenance of their internal metabolic processes, for their milk production and growth. During pregnancy, additional energy is required for the development of the foetus.

The system used in this model is the Metabolizable Energy (ME) system and the equations to estimate the energy requirements for each component are provided by MAFF (1984). An "allowance for activity" of 10% and a "safety margin" of 5% are included in all equations like in MAFF (1984).

### 5.3.1. Energy for maintenance

Energy for maintenance is the energy required for all essential life processes of a cow, such as respiration, blood circulation, walking, etc. It depends on the cow's liveweight and can be predicted by the equation

$$E_m = 8.3 + 0.091LW \tag{5.6}$$

where $E_m$ : energy required for the cow's maintenance (MJ/day)

$LW$ : cow's liveweight (kg)

Table 5.5 shows the energy for maintenance required by cows of different liveweights.

Table 5.5 - Daily energy required for maintenance

| Liveweight (kg) | 450 | 500 | 550 | 600 | 650 |
|---|---|---|---|---|---|
| Energy required for maintenance (MJ/day) | 49.25 | 53.80 | 58.35 | 62.90 | 67.45 |

## 5.3.2. Energy for pregnancy

During pregnancy cows require additional energy for the maintenance and development of the foetus. This requirement increases exponentially throughout the pregnancy and is considerably higher in the final stages of pregnancy (MAFF, 1984; NRC, 1988). Equation (5.7) estimates the energy required (MJ/day) for pregnancy:

$$E_p = 1.13\, e^{\,0.0106\, t} \tag{5.7}$$

where t : number of days pregnant.

Up to the fifth month of pregnancy the calculated energy required for pregnancy is low (less than 5 MJ/day) and can be neglected with no consequences. This assumption is in agreement with data from NRC (1988), which takes into account the energy required for pregnancy only during the last two months of gestation. ARC (1980) also presents a table with predicted energy content of the gravid uterus between days 141 to 281 of gestation.

It is assumed as default in the LP model that cows get pregnant at the beginning of the 13th week of lactation. This aims to achieve a 365-day calving interval with a 10-month lactation and an 8-week dry period.

## 5.3.3. Energy for milk production

The energy required for milk production depends on the energy value of the milk produced, which varies from one breed to another. It is arguable that nutrition does not affect the fat percentage of milk as much as genetics (Goss, 1987). *"Feed for quantity, Breed for quality."*

The metabolizable energy required for milk production is calculated from the equation (5.8), assuming that the efficiency of utilization of ME for milk production is constant at 0.62 (MAFF, 1984):

$$E_y = 1.694\left[\,0.0386\, BF + 0.0205\, SNF - 0.236\,\right] \tag{5.8}$$

where ($E_y$) is the metabolizable energy required for milk production (MJ/kg milk), and (BF) the butter fat content and (SNF) the solids non-fat content (g/kg of milk). The energy value of the milk depends on the milk quality, which is closely related to the breed.
(NOTE: BF = 10 *x* BF % and SNF = 10 *x* SNF %).

The average Friesian milk composition given by Nix (1995) is shown in Table 5.6 (Solids Non-Fat figure extracted from MAFF, 1984).

**Table 5.6 - Average milk composition of Friesian cows and energy required**

| Butter Fat (g/kg) | Solids Non-Fat (g/kg) | Energy value (MJ/kg) | ME required (MJ/kg) |
|---|---|---|---|
| 39.8 | 86 | 3.06 | 5.19 |

It is assumed in the LP model that milk density is 1.03 kg/litre (Nix, 1995).

## 5.3.4. Energy for liveweight gain and from liveweight loss

When cows intake energy above their requirements for maintenance and pregnancy, they allocate part of this surplus energy to tissue deposition and part to milk production. When they do not intake enough energy for their maintenance and pregnancy, they mobilize their body reserves to produce milk.

Body tissue has an energy value of approximately 20 MJ/kg (MAFF, 1984) and the metabolizable energy required to gain or lose weight is calculated below.

*Weight gain*

During lactation, cows use energy for body gain with the same efficiency as they use it to produce milk: 0.62 (MAFF, 1984). Hence, a gain in weight of 1 kg increases the cow's requirement for metabolizable energy (ME) by 34 MJ:

$$\frac{20}{0.62} x 1.05 \approx 34 \text{ MJ / kgLWgain}$$

This high efficiency for gain only applies to cows in lactation. Dry cows gain weight with a lower efficiency: 0.435 (MAFF, 1984). The increase of Metabolizable Energy for a dry cow to gain 1 kg is then 48 MJ:

$$\frac{20}{0.435} x 1.05 \approx 48 \text{ MJ / kgLWgain}$$

*Weight loss*

When cows are underfed in energy, they mobilize their body reserves with a subsequent weight loss. Energy mobilized from body tissues may become available for milk production with an efficiency of 0.82 (MAFF, 1984). Each kilogram of tissue mobilized will provide 16.4 MJ as milk. This is equivalent to a dietary metabolizable energy of 28 MJ, assuming an efficiency of utilization at ME at 0.62 and including safety margin, as usual (MAFF, 1984):

$$\frac{20\,x\,0.82\,x\,1.05}{0.62} \approx 28 \;\; \mathrm{MJ\,/\,kg\,LW\,loss}$$

### 5.3.5. Total energy requirements

The total amount of energy required by cows is the sum of the energy required for maintenance, pregnancy, milk production and growth. Energy from body reserves, when mobilized, has also to be taken into account in the system.

A summary of the equations used in the model to predict the metabolizable energy required for each component is shown in Table 5.7.

**Table 5.7 - Equations to predict Metabolizable Energy required by dairy cows**

| | Equation | Unit |
|---|---|---|
| Maintenance ($E_m$) | 8.3 + 0.091 LW | MJ/day |
| Pregnancy ($E_p$) | $1.13\ e^{0.0106\,t}$ | MJ/day |
| Energy Value of Milk ($EV_y$) | 0.0386 BF + 0.0205 SNF - 0.236 | MJ/kg milk |
| Milk production ($E_y$) | 1.694 $EV_y$ | MJ/kg milk |
| Body weight gain ($E_w^+$) | | |
| lactating cows | 34 | MJ/kg LW gain |
| dry cows | 48 | MJ/kg LW gain |
| Body weight loss ($E_w^-$) | 28 | MJ/kg LW loss |

*LW* : liveweight (kg),
*SNF* : solids non-fat content (g/kg)
*BF* : Butter fat content (g/kg),
t : number of days pregnant.

## 5.4. Grass crop

The pattern of grass growth varies throughout the season with a marked peak occurring during the phase of stem elongation in May/June (Green *et al*, 1971; Audsley, 1974). Rapid changes in the yield and in the digestibility occur. The stage when grass is cut or grazed has a great influence on the quality and on the annual yield: actually, the later it is cut or grazed the higher the yield (up to a certain limit) and the lower the quality of the grass. Hence, the sequence of cutting and grazing and the times when they are performed have an important role in determining the amount of grass available as well as its quality.

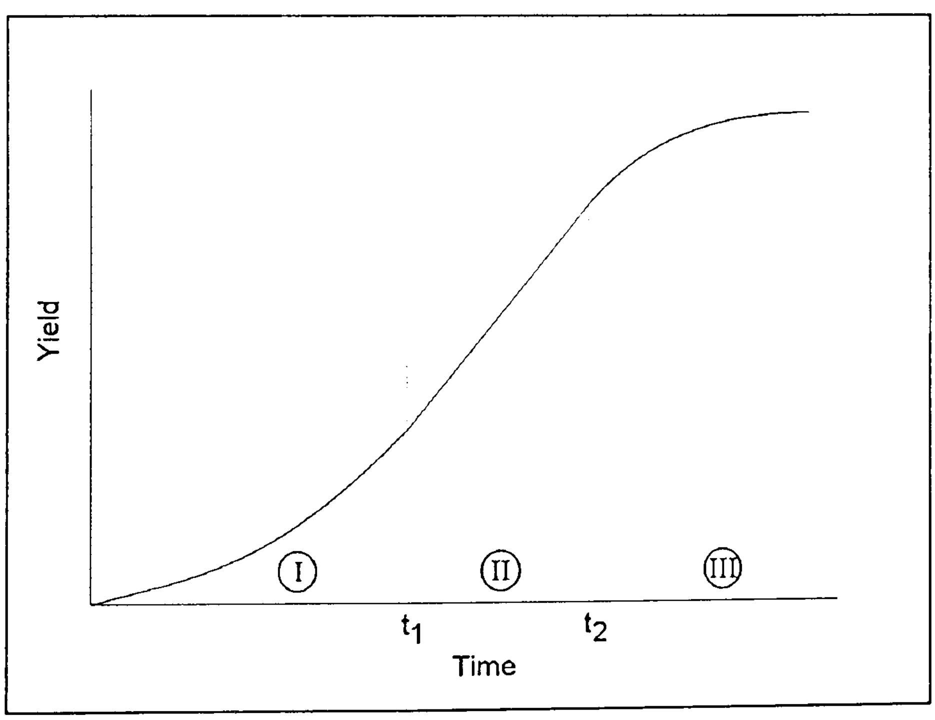
In order to determine the optimum grassland use, data concerning grass yield and digestibility when cut at irregular intervals is necessary. Experiments to examine these situations are difficult and costly, since there are many different possible sequences of cutting intervals (Pohjonen, 1975; Edelsten and Corrall, 1979). The high variability of environmental conditions is another difficulty, for it has a strong influence on the productivity (Woodward, 1993).

### 5.4.1. Yield

There are many ways of predicting grass yield ranging from mechanistic models to regression models and actual measured data. Some important factors are nitrogen (N) application rate, soil type, expected rainfall and irrigation.

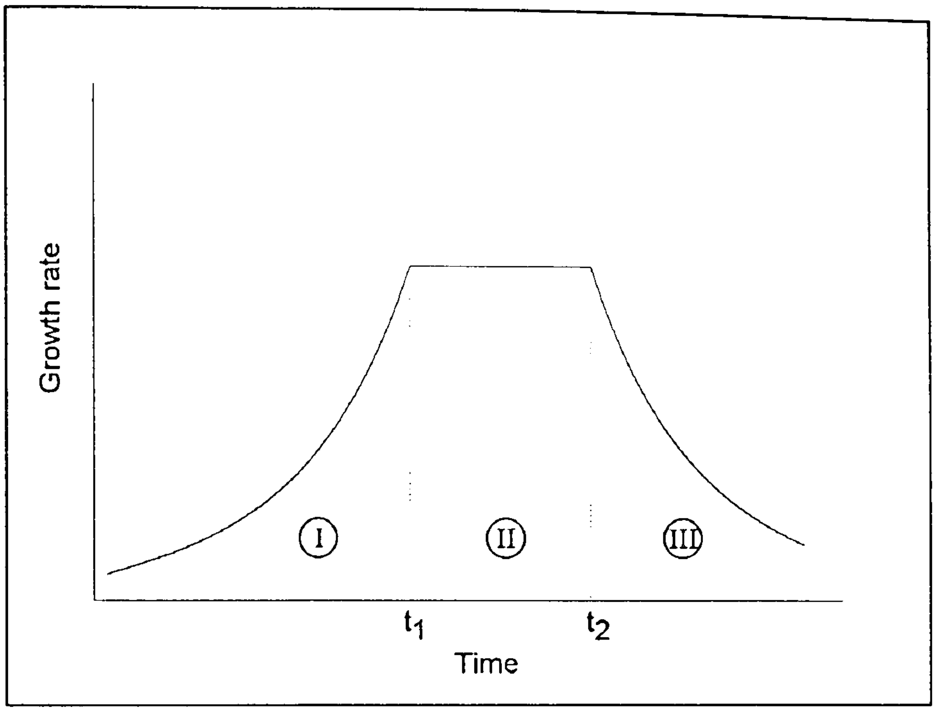
Edelsten and Corrall (1979) used data from experiments carried out at Grassland Research Institute (GRI), which cover a wide range of cutting sequences with a reduced set of treatments. They constructed regression models capable of providing estimates of yield under any given sequence of cuts. Their model assumes that the grass follows a logistic growth curve as shown in Figure 5.3.

**Figure 5.3 - Logistic growth curve used by Edelsten and Corrall (1979)**




The yield curve has the following pattern: a period of exponential rise in growth rate followed by a period in which the rate is constant at a 'maximum' level and then a period in which the rate declines exponentially. Figure 5.4 shows the growth rate curves. Part I concerns the exponential phase of grass growth, Part II the linear phase and Part III the asymptotic phase.

Figure 5.4 - Growth rate curve used by Edelsten and Corrall (1979)




The model assumes that the maximum yield rate varies throughout the year ($g_i$) and is reduced by a factor ($r_i$), as shown in equation (5.9).

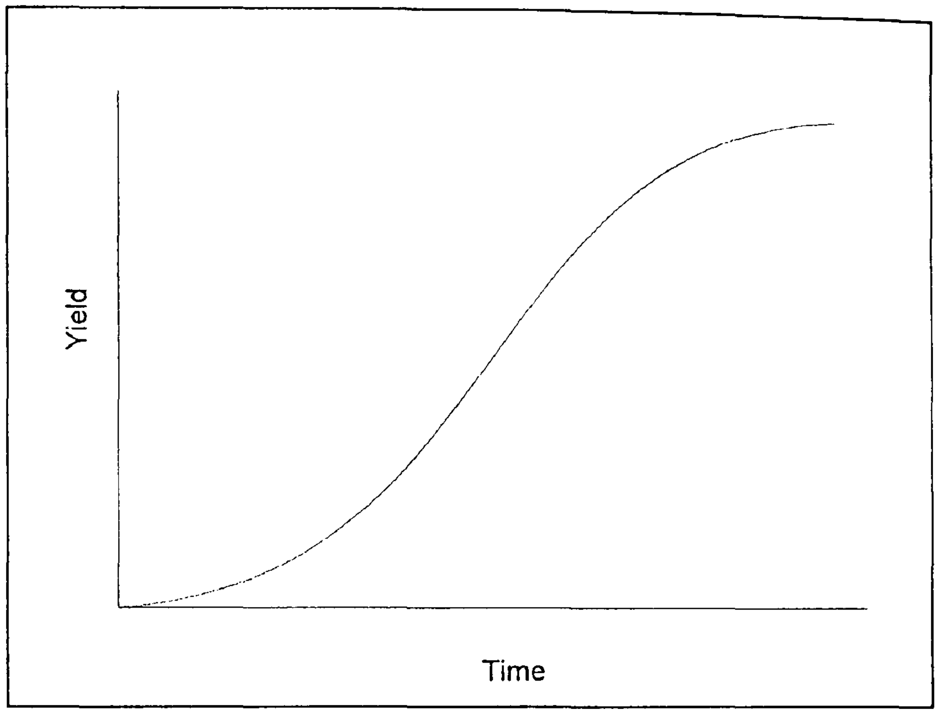
$$y = \sum_{i=t_0}^{t} g_i r_i \tag{5.9}$$

where $t_0$ is the time when plot was last cut
$y$ is the yield available for cutting or grazing

$$r_i = \begin{cases} e^{-\beta(t_1-i)}, & \text{during exponential phase} \\ 1, & \text{during linear phase} \\ e^{-\beta(i-t_2)}, & \text{during asymptotic phase} \end{cases}$$

Parsons (1992a) presents a reasonably simple grass model capable of predicting yield which assumes that the grass follows a logistic growth curve, shown in Figure 5.5, but with only two phases: exponential and asymptotic.

**Figure 5.5 - Logistic growth curve used by Parsons (1992a)**

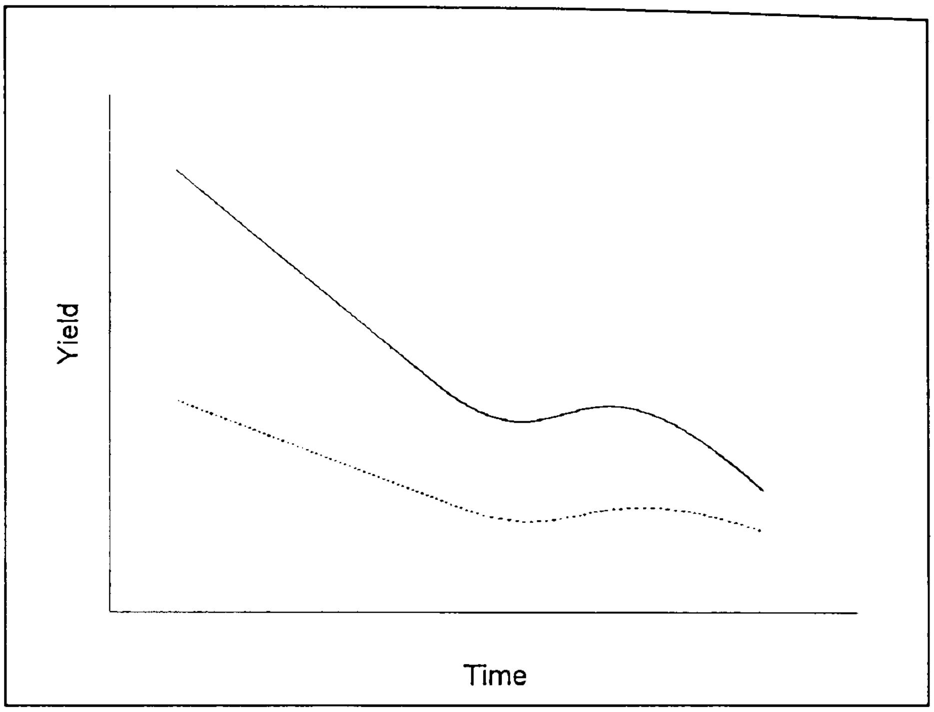



Equation (5.10) determines the grass yield with parameters (*b*) and (*c*) determining the shape and the asymptote of the curve, respectively, and (*a*) and (*m*) performing simple translations.

$$y = a + \frac{c}{1 + e^{b(t-m)}} \tag{5.10}$$

Once a field plot has been cut, grass restarts to grow following a similar pattern, with parameter (*c*) varying during the season. A family of regrowth curves for different intervals of regrowth is shown in Figure 5.6. The pattern of regrowth provides a 'second peak' that is observed in many silage-making experiments.

**Figure 5.6 - Family of regrowth curves used by Parsons (1992a)**




Equation (5.11) predicts the grass yield after an interval time for regrowth:

$$y = a + \frac{c}{1 + e^{b(t_r - m)}} \tag{5.11}$$

where $c$=1.617x10$^{-9}$ $t_c^5$–1.4391x10$^{-6}$ $t_c^4$+4.9378x10$^{-4}$ $t_c^3$–0.08105 $t_c^2$+6.2809 $t_c$–176.54

$t_c$ : date when plot is cut (Julian day; Jan 1st = 1)

$t_r$ : interval time for regrowth (days)

In this simulation model the grass growth is modified according to nitrogen level, soil moisture and temperature. Despite some limitations of this model, it gives acceptable predictions for yield and D-value.

## 5.4.2. Digestibility

Edelsten and Corrall (1979) also present a model to predict the digestibility of grass. The model was constructed with a linear regression of *in vitro* digestibility data from an experiment carried out at GRI. The amount harvested, the interval time since the previous cut and the time of the year were established as significant factors affecting digestibility. They were incorporated into a linear regression equation.

$$D_v = 73.6 + 2.5 sin\omega + 1.6 cos\omega - 0.108t - 0.715y \tag{5.12}$$

where $D_v$ is the digestible organic matter content (D-value), in %

$\omega = 2 \pi i / 365$ ( $i$ = cut date — Julian day )

$t$ = interval time since previous cut (days)

$y$ = amount harvested (tDM/ha)

## 5.5. Grass silage

Grass silage is the main ration for dairy cows during winter. When properly made, grass silage is likely to be digestible and of high nutritive value characteristics. Feeding value of the silage and efficiency of its conservation are strongly related; together, they define the quality of the silage. Good conservation, with minimum losses, usually leads to a higher feeding value, which is determined mainly by the nutrient content of the grass ensiled.

### 5.5.1. Silage losses

Losses occur during the various phases of the silage-making process. Most authors conveniently divide them into two main groups: field losses and in-silo losses. Many papers about losses during conservation of grass forage have been published. Some are based on experiments (Mayne and Gordon, 1986a; Mayne and Gordon, 1986b; Bastiman and Altman, 1985) and some are extensive reviews analysing and comparing experimental results with mathematical models (McGechan, 1989; McGechan, 1990a).

#### 5.5.1.1. Field losses

Field losses can be divided into respiratory losses, losses due to leaching by rain and losses caused by mechanical treatments.

<u>Respiratory losses</u>

Field losses reported by Bastiman and Altman (1985) average 4.8 % DM for wilted silage. Since these field losses were estimated under good wilting conditions (rain-free days) and with no mechanical treatments, it is assumed that these losses are mostly due to respiratory loss. This value is reasonable when compared with respiratory losses shown in McGechan (1989), when wilting period varies from 2 to 3 days.

<u>Losses due to leaching by rain</u>

Lack of information concerning losses due to leaching by rain reveals the difficulty of assessment. Experimental results are mainly for the sum of respiratory plus leaching losses, the latter being estimated by subtracting the respiratory loss measured during rain-free days (McGechan, 1989). It is reasonable to assume low losses (1 to 2 % DM) due to leaching by rain provided wilting is done under good weather conditions (1 to 2 mm rainfall).

Longer wilting periods to achieve higher dry matter contents are more risky and susceptible to higher losses due to leaching by rain (Bastiman and Altman, 1985).

*Mechanical losses*

Mechanical losses are mainly due to fragmentation of grass; these fragments drop on the field or are blown away, not being picked up by the forage harvester. This type of loss occurs during mechanical operations (e.g., mowing and picking up).

Direct cut harvesters perform fewer operations than precision chop harvesters and consequently produce lower mechanical losses. Nonetheless, other losses occur when grass is gathered with direct cut harvesters (e.g., effluent losses and losses due to poorer fermentation).

It is assumed that pick-up losses are independent of yield or moisture content, depending mainly on the machine and treatment performed (e.g., wilted or unwilted) and figures vary from 0.6 % DM to 1.5 % DM (McGechan, 1990b).

Mayne and Gordon (1986a) found average total mechanical losses varying from 0.8 % DM (flail forage harvester, unwilted) to 2.2 % DM (precision chop harvester, wilted), with an intermediary loss of 2.0 % DM (precision chop harvester, unwilted). These losses include pick-up losses and losses during other operations (e.g., tedding and cutting).

#### 5.5.1.2. In-silo losses

Losses occurring during storage of silage are particularly large and depend on various factors such as the nutrient content of the grass ensiled, moisture content, chop length and whether additive has been used or not. Once ensiled, complex biochemical processes start and considerable changes occur in the cut grass, producing losses of nutrients. These losses are due to plant respiration just after ensiling, fermentation, and losses from the effluent released by silage made with grass with low dry matter content. Therefore, in-silo losses can be conveniently divided into losses due to air infiltration (during filling and feed-out), fermentation and effluent loss.

*Losses due to air infiltration*

These losses may occur during the period of filling silos, the storage period and the period that silage is being fed out.

Losses due to respiration are usually small when air-free conditions are quickly achieved in the silo. This reduces the amount of oxygen available for the oxidation of the water soluble carbohydrates (WSC) of the grass and allows a lactic acid fermentation to occur soon producing a stable silage.

In many experiments, invisible losses have been measured by comparing of the weight of grass put into and taken out of silos. Invisible losses are calculated subtracting spoilt material and effluent from that difference. However, it is not possible to know which loss is due to air infiltration and which loss is due to fermentation. Losses due to respiration during the storage period are usually not measured and are more commonly included in the "invisible losses". McGechan (1990b) suggests values of losses due to air infiltration during filling silos between 1 % DM (with additives) and 2 % DM (no additives).

Losses during the period that silage is being fed out are due to aerobic deterioration. After a silo is opened oxidation of the nutrients occurs producing loss. McGechan (1990a) presents some values for aerobic deterioration losses during the feed out period with different treatments. There is a consistent relationship between aerobic deterioration and the D-value of the silage: the higher the D-value, the lower the loss. When applying standard rates of additive, typical values for these losses vary from 3 % DM (precision chop, 20% DM, direct cut, 70 % D-value) to 13 % DM (precision chop, 30% DM, direct cut, 60 % D-value).

It is generally accepted that the surface waste produced during the storage results in nutrient losses. McGechan (1990b) suggests losses due to surface waste varying from 8 % DM (precision chop harvester, direct cut) to 12 % DM (flail harvester, wilted). This is in agreement with the theory that short-chopped grass enables a higher density to be achieved and consequently a higher resistance to oxygen ingress into the silage during storage, reducing surface waste losses (Mayne and Gordon, 1986b).

*Losses due to fermentation*

Losses due to fermentation are usually given as "invisible losses", which also includes losses due to respiration during the storage period and after the silo is opened. "Invisible losses" have been reported in several experiments and vary widely since they depend on several factors such as type of silo, chop length of grass ensiled and efficiency of preservation.

When air-free conditions are quickly achieved in the silo, a lactic acid fermentation starts early, lowering the pH of the silage. This prevents undesirable bacterias (e.g., clostridia) from becoming active, avoiding lactic acid being converted to butyric acid, which has an undesirable smell and reduces the voluntary intake of silage by ruminants.

McGechan (1990a) presents several values of total invisible losses for different treatments and forage harvesters. In general, invisible losses (due to fermentation plus air infiltration during storage) increased with the D-value of the silage and decreased with the percentage of dry matter of the grass ensiled. For standard rates of additives and precision chop harvesters, typical values for total invisible losses varied from 3 % (30% DM, 60% D-value) to 9 % DM (20% DM, 70% D-value). For flail harvesters, they varied from 7 % DM (30% DM, 60% D-value) to 14 % DM (20% DM, 70% D-value).

In the LP model it is assumed that silage will contain additive (applied at a standard rate) and that the silage will be properly made. It is reasonable to assume "invisible loss" due to air infiltration plus fermentation at 7 % DM.

*Effluent losses*

The volume of effluent produced by silage is mainly dependent on the moisture content of the grass ensiled, although the size and type of silo also have some influence. Bastiman and Altman (1985) found that the average dry matter content of the effluent was 5.8 % and was not related to the amount of effluent produced. The DM losses in the effluent were related only to the amount of effluent produced and determined by the DM content of the grass ensiled. They found %DM losses in the effluent varying from 6.5% to 2.0% to 0.5% for grass ensiled at 15 %DM, 20% DM and 25 %DM, respectively. Generally when %DM of the grass to be ensiled is above 30%, the dry matter loss in effluent is negligible (McGechan, 1990a; Bastiman and Altman, 1985; Donaldson, 1984).

### 5.5.1.3. Total dry matter losses and estimate of digestible nutrients

Dry matter losses during the various phases of the silage-making process have already been discussed so it remains to see how the model estimates the digestible matter of the silage. DOMD (or D-value) is the content of digestible organic matter in dry matter of the silage. It measures the digestibility of the silage and consequently its quality. Although the quality of the silage is strongly dependent on the harvested grass, it may be seriously affected by the loss of nutrients during the various phases of the silage-making process. Some of those losses previously discussed reduce both the non-digestible and the digestible components of the silage (e.g., mechanical losses) and some reduce mainly (or only) the digestible part of the silage (e.g., respiration, fermentation and losses due to leaching by rain).

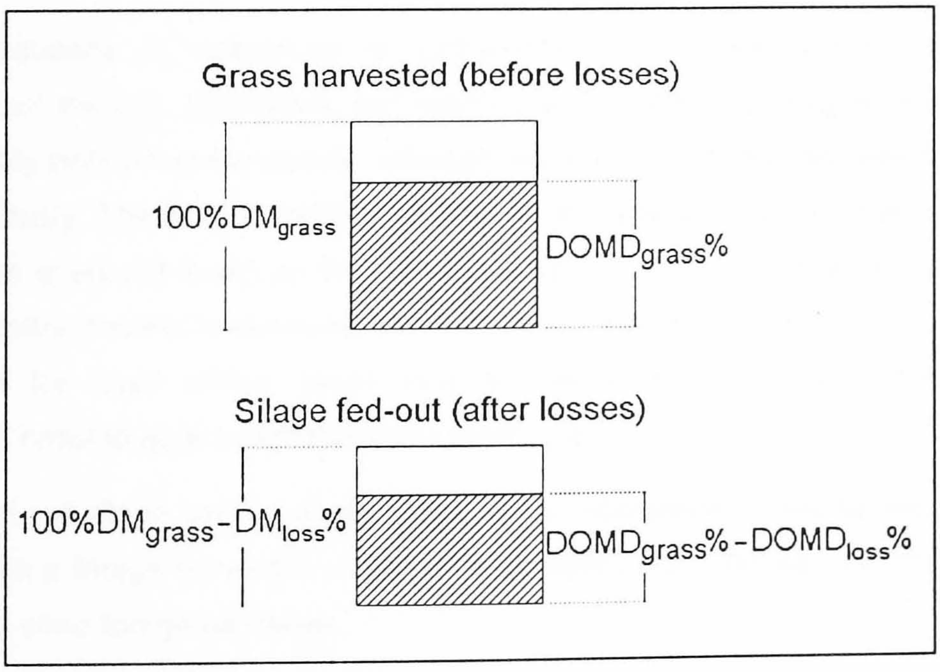
For example, let us assume that grass is ensiled with 25 %DM (after wilting). Dry matter losses of the silage, as previously discussed, are summarized in Table 5.8 with an estimate of the proportion of the digestible component lost in each category.

**Table 5.8 - Dry matter losses and estimates of proportion of digestible component lost**

| | DM loss % | % of digestible nutrients | DOMD loss % |
|---|---|---|---|
| **Field losses** | | | |
| Respiration | 4.8 | 100 | 4.8 |
| Leaching by rain | 1.0 | 100 | 1.0 |
| Mechanical treatment | 1.5 | 50 | 0.75 |
| **In-silo losses** | | | |
| Air-infiltration during filling | 1.0 | 100 | 1.0 |
| Air-infiltration during feed-out (includes surface waste) | 8.0 | 100 | 8.0 |
| Invisible losses | 7.0 | 50 | 3.5 |
| Effluent | 0.5 | 50 | 0.25 |
| Total | 23.8 | | 19.3 |

Once the estimated total dry matter loss ($DM_{loss}$%) and the estimated total digestible organic matter loss ($DOMD_{loss}$%) are known, it is possible to estimate the DOMD of the silage after losses. Figure 5.7 illustrates the dry matter loss ($Dm_{loss}$) and the digestible organic matter loss ($DOMD_{loss}$) in the silo.

**Figure 5.7 - DM and DOMD losses in the silo**




The percentage of digestible organic matter of the silage after losses ($DOMD_{silage}$%) can be calculated according to the equation (5.13):

$$DOMD_{silage}\% = 100\left(\frac{DOMD_{grass}\% - DOMD_{loss}\%}{100 - DM_{loss}\%}\right) \quad (5.13)$$

where $DOMD_{grass}\%$ : digestible organic matter of the grass harvested (before losses).

In the LP model the default total dry matter loss ($DM_{loss}\%$) is 23.8% and the total digestible organic matter loss ($DOMD_{loss}\%$) is 18.8% (see Table 5.8).

Typical values for $DOMD_{silage}\%$ after losses (using default data for losses) are shown in Table 5.9, for a range of $DOMD_{grass}\%$ varying from 60% to 74%.

**Table 5.9 - Typical values of DOMD% and predicted metabolizable energy (ME) of silages**

| $DOMD_{grass}\%$ | $DOMD_{silage}\%$ | ME (MJ/kgDM) (*) |
|---|---|---|
| 60 | 54.1 | 8.61 |
| 62 | 56.7 | 9.04 |
| 64 | 59.3 | 9.48 |
| 66 | 61.9 | 9.91 |
| 68 | 64.6 | 10.34 |
| 70 | 67.2 | 10.78 |
| 72 | 69.8 | 11.21 |
| 74 | 72.4 | 11.64 |

(*) ME = 0.16 DOMD% (MAFF, 1984)

### 5.5.2. Field operations

A sequence of operations is performed during grass conservation: mowing, windrowing (not always), harvesting, transporting and ensiling. Mowing is the first operation and it is usually performed separately, although there are some machines that mow and pick up simultaneously. The next operation depends on the type of silage system. If wilted silage is desired, the grass cut is left on the field (in good weather) for a certain period or until a certain dry matter content is achieved, when it is then harvested. When grass is mowed into small swaths for rapid wilting, these may be windrowed into one immediately before harvesting, in order to give a high harvester work rate.

Direct cut silage can be made either in two operations – cutting with a mower and picking up with a forage harvester – or in a single operation – cutting and picking up with a flail or double chop forage harvester.

Silage losses are strongly affected by the type of silage chosen: wilted or unwilted. Wilted grass silages have higher field losses (but lower in-silo losses) than unwilted. Work rate and costs are also affected by the type of silage, as one or two operations are performed for unwilted or wilted silages, respectively.

The last operations performed are transporting the silage from the field to the clamp and ensiling it into the clamp. Transport is dependent on the harvester work rate and the number of trailers available, trailer size, grass density and distance from the field to the clamp.

There is usually a tractor available with sufficient power for most conservation operations (e.g., mowing, windrowing and transport). Work rates for these operations are usually limited by the maximum tractor forward safe speeds (McGechan, 1986). Forage harvesting is unique in that it has the largest power requirement when compared to any other operation. It is performed at the maximum work rate possible with the tractor available. When grass yield is low (e.g., second and third cuts), work rate might be limited not by tractor power but by maximum harvester speed.

#### 5.5.2.1. Combination of machines

Information about several combinations of machines for forage conservation and work rates can be obtained from literature (Nix, 1995; McGechan, 1986; ABC, 1993).

Precision chop harvesters have become more popular in UK, achieving more than 70% of the market. Catt (1984) and recent sales statistics confirm this popularity.

A typical machinery set to make wilted silage is shown in Table 5.10. Further details of different possible machine combinations and systems (with respective efficiencies) can be found in McGechan (1986).

**Table 5.10 - Combination of machines to perform field operations during silage-making**

| Operation | Implement | Tractor size required |
|---|---|---|
| mowing | 2.1 m drum mower-conditioner | 65 kW |
| harvesting | precision chop forage harvester and silage trailer (6 t) | 75 kW |
| transporting | second silage trailer (6 t) | 45 kW |
| filling clamp | buckrake | 45 kW |

The cutting operation is performed separately when wilted silage is made. The grass is cut and left on the field either until achieving a desired percentage of dry matter content or for a specific period. For the set shown above, three tractors would be required. and the same tractor used for mowing can be used to fill the clamp. The density of the grass ensiled is approximately 150 kgDM/m$^3$, estimated from McGechan (1990a) (25% DM and 65% D-value).

#### 5.5.2.2. Forward speed

Based on the extensive review on forage chopping by O'Dogherty (1982), forward speed of a precision chop harvester can be calculated by the following relationship, described in McGechan (1986) and McGechan (1990b):

$$\begin{cases} 0.15 G s + \left(1.82 \dfrac{\%DM}{l} + 3.02\right) \dfrac{y_w w s}{36} + 2.73 G s \tan\alpha + 4 < P \\ s < s_{max} \end{cases} \tag{5.14}$$

where $G$ : 8.2 t for a 75 kW tractor plus harvester side loading or 15.2 t for a 3-in-line combination with full trailer

P : tractor power (kW)

$s$ : forward speed (km/h)

$s_{max}$ : maximum forward speed (km/h)

$\%DM$ : dry matter content of grass after wilting (%)

$l$ : chop length (mm)

$y_w$ : wet yield after wilting (t/ha)

$w$ : picking up width (m)

$\alpha$ : slope (°)

Equation (5.14) includes a maximum forage harvester speed for lower yield fields (e.g., second and third cuts).

Power requirement, at a certain speed, is proportional to the system weight, width of the pick up device, amount of grass to be harvested and its dry matter content. In contrast, the power requirement is inversely proportional to the chop length.

### 5.5.2.3. Workrate

Harvester workrate in a certain field can be calculated from the forward speed previously calculated by equation (5.15).

$$r = \left[ \frac{s\,w}{10} + \frac{V\,\rho}{t\,\%DM\,y_w} \right] \eta \tag{5.15}$$

where
- $r$ : workrate (ha/h)
- $s$ : speed (km/h)
- $w$ : pick up width (m)
- V : total volume of trailers ($m^3$)
- $\rho$ : ensiled grass density ($tDM/m^3$)
- $t$ : trailer change time (h)
- $\%DM$ : dry matter content of grass after wilting (%)
- $y_w$ : wet yield after wilting (t/ha)
- $\eta$ : field efficiency (70% for side loading trailer and 66% for 3-in-line)

### 5.5.2.4. Tractor fuel consumption

Tractor fuel consumption at maximum power assumed in the LP model is 0.344 litres/kWh, which represents a thermal efficiency of 29.1% (McGechan, 1990c). Average tractor fuel consumption has been taken as 40% of its rated power for all operations, except harvesting, as assumed by Corrall *et al* (1983) and used by McGechan (1990c).

When calculating tractor fuel consumption during forage harvesting, tractor power is the rated tractor power if forward speed is below $s_{max}$ (reflecting that the limitation is the tractor power available). When forward speed is calculated to be above $s_{max}$,, the power required to work at the maximum speed is calculated and the fuel consumption is calculated from this power requirement (reflecting that the maximum forward speed of the forage harvester is limiting the work rate).

Table 5.11 shows the tractor fuel consumption for cutting, harvesting, transporting and filling clamp operations of the machinery set of Table 5.10.

**Table 5.11 - Tractor fuel consumption for cutting, harvesting, transporting and filling clamp operations**

| Operation | Tractor size (kW) | Fuel consumption (litre/hour) |
|---|---|---|
| cutting | 65 | 0.4 x 65 x 0.344 = 8.944 |
| transporting | 45 | 0.4 x 45 x 0.344 = 6.192 |
| filling clamp | 65 | 0.4 x 65 x 0.344 = 8.944 |
| harvesting | 75 | |
| | when speed $\leq s_{max}$ | 75 x 0.344 = 25.8 |
| | when speed = $s_{max}$ | $P_{actual}$ x 0.344 |

$$P_{actual} = s_{\max}\left[0.15\,G + \left(\frac{1.82\ \%DM}{l} + 3.02\right)\frac{y_w\,w}{36} + 2.73\,G\tan\alpha + 4\right]$$

## 5.6. Maize silage

Higher summer milk prices attract farmers to summer calving and maize silage has proved an excellent complement to summer grazing. For these reasons there has been an increase in the popularity of the maize crop on dairy farms in UK.

Maize silage is made during the Autumn and usually fed in the following Winter, Spring and Summer. Cows fed a mixture of maize and grass silages not only consume less concentrates but also respond better to this mixture than to grass alone. Milk production is increased and even an alteration of the fat and protein content of the milk occurs.

Furthermore, when compared to grass silage, maize is cheaper to produce, demands much lower fertilizers and chemicals and produces a much higher dry matter content silage than grass.

Table 5.12 shows typical values of maize crop and maize silage in UK (Nix, 1995; Kingshay, 1994).

**Table 5.12 - Maize silage: fresh yield, dry matter content, dry matter loss, D-value and Metabolizable Energy**

| Fresh yield (t/ha) | %DM | %DM loss | D-value (%) | ME (MJ/kgDM) |
|---|---|---|---|---|
| 40 | 33 | 20 | 70 | 11.2 |

When cows are fed large amounts of good quality grass silage, it is reasonable to assume that all their protein requirements (mainly rumen degradable protein – RDP) will be provided. However, things change when maize silage is added to the ration, because it has a high energy value, but low protein content. When maize silage is fed in a high proportion, some complementary feed, rich in protein, might be necessary. The model limits the percentage of maize silage in the ration. This limitation is provided by the user and will depend on the protein content of the other feeds available.

## 5.7. Concentrates

A reduction of forage intake occurs when concentrates are fed with forage and this reduction is known as the "substitution effect". Although it is a well-known phenomenon, it has not been included in major systems that calculate nutrient requirements for dairy cows (Hulme et al, 1986). The "substitution effect" is taken into account in this LP model.

When forage is fed alone, dry matter intake is usually lower than when fed with concentrates. The amount of concentrates eaten reduces the intake of forage by a smaller amount; for instance, 1 kg of concentrates eaten reduces the intake of forage by less than 1 kg. The total dry matter intake will be higher when concentrates are fed in conjunction with forage.

In order to illustrate the "substitution effect" in the total dry matter intake, a numerical example is given below (assuming that 1 kg DM of concentrates eaten reduces 0.8 kg DM of forage intake):

| Concentrates (kgDM) | Forage (kgDM) | Total Intake (kgDM) |
|---|---|---|
| 0 | 16.0 | 16.0 |
| 1 | 15.2 | 16.2 |
| 2 | 14.4 | 16.4 |
| 3 | 13.6 | 16.6 |
| 4 | 12.8 | 16.8 |

Hulme *et al* (1986) state that the rate at which concentrates substitute for forage varies with the proportion of concentrates in the diet. The following reductions in forage intake per kg of concentrate eaten are suggested:

- 0.64 kg when proportion of concentrates is less than 25% of the total ration.
- 0.84 kg when proportion of concentrates is between 25% and 50% of the total ration.
- 1.22 kg when proportion of concentrates is more than 50% of the total ration.

In this LP model, the reduction due to "substitution effect" is assumed to be constant regardless of the proportion of the concentrate in the ration. It is also assumed that cows are fed at least 1 kg/day of concentrates during the four weeks before calving. This is called "steaming up" and aims to allow the cow and the rumen to become adapted to a diet similar to that one required in early lactation (NRC, 1988).

## 5.8. Summary

The models discussed in this chapter are calculated for each period and put together by a computer program which generates the LP matrix.

In order to show how all data is fitted in the LP, an example is given below for cows calving in Period 1 (January).

| | $a_c$ | $a_z$ | $g_{1,5,0}$ | $g_{1,6,0}$ | … | $g_{1,9,0}$ | $g_{1,7,2}$ | $g_{1,8,2}$ | $g_{1,8,3}$ | … | $s_{9,0}$ | $s_{10,0}$ | … | $s_{10,5}$ | $s_{11,5}$ | $s_{11,6}$ | $s_{12,5}$ | $s_{12,6}$ | $s_{12,7}$ | … | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Area | 1 | 1 | 1 | 1 | | 1 | | | | | 1 | 1 | | | | | | | | | = | 70 |
| Land 5 | | | -1 | | | | 1 | | 1 | | | | | 1 | | 1 | | | 1 | | = | 0 |
| Land 6 | | | | -1 | | | | 1 | | | | | | | 1 | | | 1 | | | = | 0 |
| Land 7 | | | | | | | -1 | | | | | | | | | | 1 | | | | = | 0 |
| Land 8 | | | | | ⋱ | | | -1 | -1 | | | | | | | | | | | | = | 0 |
| Land 9 | | | | | | -1 | | | | | | | | | | | | | | | = | 0 |
| Land10 | | | | | | | | | | | -1 | | | | | | | | | | = | 0 |
| Land11 | | | | | | | | | | | | -1 | | -1 | | | | | | | = | 0 |
| ⋮ | | | | | | | | | | | | | | | | | | | | | ⋮ | |
| HrvTime1 | | | | | | | | | | | 1.10 | 1.16 | | | | | | | | | ≤ | 0 |
| HrvTime2 | | | | | | | | | | | | | | 1.07 | 1.07 | 1.09 | 1.06 | 1.09 | 1.10 | | ≤ | 0 |
| DM 1 1 | | | | | | | | | | | | | | | | | | | | | ≤ | 0 |
| DM 1 2 | | | | | | | | | | | | | | | | | | | | | ≤ | 0 |
| DM 1 3 | | | | | | | | | | | | | | | | | | | | | ≤ | 0 |
| DM 1 4 | | | | | | | | | | | | | | | | | | | | | ≤ | 0 |
| DM 1 5 | | | 0.94 | | | | | | | | | | | | | | | | | | ≤ | 0 |
| DM 1 6 | | | | 1.58 | | | | | | | | | | | | | | | | | ≤ | 0 |
| DM 1 7 | | | | | | | 0.51 | | | | | | | | | | | | | | ≤ | 0 |
| DM 1 8 | | | | | ⋱ | | | 0.44 | 1.05 | | | | | | | | | | | | ≤ | 0 |
| DM 1 9 | | | | | | 3.66 | | | | | | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | | | | | | | ⋮ | |
| ME 1 1 | | | | | | | | | | | | | | | | | | | | | = | 0 |
| ME 1 2 | | | | | | | | | | | | | | | | | | | | | = | 0 |
| ME 1 3 | | | | | | | | | | | | | | | | | | | | | = | 0 |
| ME 1 4 | | | | | | | | | | | | | | | | | | | | | = | 0 |
| ME 1 5 | | | -11.3 | | | | | | | | | | | | | | | | | | = | 0 |
| ME 1 6 | | | | -18.9 | | | | | | | | | | | | | | | | | = | 0 |
| ME 1 7 | | | | | | | -6.3 | | | | | | | | | | | | | | = | 0 |
| ME 1 8 | | | | | ⋱ | | | -5.3 | -12.5 | | | | | | | | | | | | = | 0 |
| ME 1 9 | | | | | | -39.1 | | | | | | | | | | | | | | | = | 0 |
| ⋮ | | | | | | | | | | | | | | | | | | | | | ⋮ | |
| MY 1 1 | | | | | | | | | | | | | | | | | | | | | = | 0 |
| MY 1 2 | | | | | | | | | | | | | | | | | | | | | = | 0 |
| MY 1 3 | | | | | | | | | | | | | | | | | | | | | = | 0 |
| ⋮ | | | | | | | | | | | | | | | | | | | | | ⋮ | |
| CPR 1 5 | | | -0.47 | | | | | | | | | | | | | | | | | | ≤ | 0 |
| CPR 1 6 | | | | -0.79 | | | | | | | | | | | | | | | | | ≤ | 0 |
| CPR 1 7 | | | | | | | -0.26 | | | | | | | | | | | | | | ≤ | 0 |
| CPR 1 8 | | | | | ⋱ | | | -0.22 | -0.52 | | | | | | | | | | | | ≤ | 0 |
| CPR 1 9 | | | | | | -1.83 | | | | | | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | | | | | | | ⋮ | |

| | $a_c$ | $a_z$ | $g_{1,5,0}$ | $g_{1,6,0}$ | … | $g_{1,9,0}$ | $g_{1,7,2}$ | $g_{1,8,2}$ | $g_{1,8,3}$ | … | $s_{9,0}$ | $s_{10,0}$ | … | $s_{10,5}$ | $s_{11,5}$ | $s_{11,6}$ | $s_{12,5}$ | $s_{12,6}$ | $s_{12,7}$ | … | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MzPR 1 5 | | | -0.28 | | | | | | | | | | | | | | | | | | ≤ | 0 |
| MzPR 1 6 | | | | -0.47 | | | | | | | | | | | | | | | | | ≤ | 0 |
| MzPR 1 7 | | | | | | | -0.15 | | | | | | | | | | | | | | ≤ | 0 |
| MzPR 1 8 | | | | | ⋱ | | | -0.13 | -0.31 | | | | | | | | | | | | ≤ | 0 |
| MzPR 1 9 | | | | | | -1.10 | | | | | | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | | | | | | | ⋮ | |
| DMin1 | | | | | | | | | | | 4.6 | 5.7 | | | | | | | | | = | 0 |
| DMin2 | | | | | | | | | | | | | | 3.2 | 2.9 | 4.1 | 2.6 | 3.8 | 4.6 | | = | 0 |
| DMout1 | | | | | | | | | | | | | | | | | | | | | ≤ | 0 |
| DMout2 | | | | | | | | | | | | | | | | | | | | | ≤ | 0 |
| MEin1 | | | | | | | | | | | 46.2 | 52.8 | | | | | | | | | = | 0 |
| MEin2 | | | | | | | | | | | | | | 31.5 | 28.2 | 36.8 | 24.7 | 34.1 | 36.6 | | = | 0 |
| MzBank | | -10.5 | | | | | | | | | | | | | | | | | | | ≤ | 0 |
| MxLoss1 | | | | | | | | | | | | | | | | | | | | | ≤ | 0 |
| LWbal1 | | | | | | | | | | | | | | | | | | | | | = | 0 |
| MxCh 1 1 | | | | | | | | | | | | | | | | | | | | | ≤ | 0 |
| MxCh 1 2 | | | | | | | | | | | | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | | | | | | | ⋮ | |
| Quota | | | | | | | | | | | | | | | | | | | | | ≤ | 875 000 |
| NetMrgn | 400 | 330 | -135 | -135 | … | -135 | | | | | -284 | -289 | | -56 | -55 | -60 | -54 | -59 | -62 | | = | z |

| | COW 1 | $\Delta w^+$ 1 1 | $\Delta w^+$ 1 2 | … | $\Delta w^-$ 1 1 | $\Delta w^-$ 1 2 | … | Milk 1 1 | Milk 1 2 | Milk 1 3 | … | GSlg 1 1 1 | GSlg 1 2 1 | GSlg 1 3 1 | … | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Area | | | | | | | | | | | | | | | | = | 70 |
| Land 5 | | | | | | | | | | | | | | | | = | 0 |
| Land 6 | | | | | | | | | | | | | | | | = | 0 |
| Land 7 | | | | | | | | | | | | | | | | = | 0 |
| Land 8 | | | | | | | | | | | | | | | | = | 0 |
| Land 9 | | | | | | | | | | | | | | | | = | 0 |
| Land10 | | | | | | | | | | | | | | | | = | 0 |
| Land11 | | | | | | | | | | | | | | | | = | 0 |
| ⋮ | | | | | | | | | | | | | | | | ⋮ | |
| HrvTime1 | | | | | | | | | | | | | | | | ≤ | 0 |
| HrvTime2 | | | | | | | | | | | | | | | | ≤ | 0 |
| DM 1 1 | -387 | -0.73 | | | 0.81 | | | | | | | 1 | | | | ≤ | 0 |
| DM 1 2 | -424 | -1.16 | -0.73 | | 1.29 | 0.81 | | | | | | | 1 | | | ≤ | 0 |
| DM 1 3 | -445 | -1.16 | -1.16 | | 1.29 | 1.29 | | | | | | | | 1 | | ≤ | 0 |
| DM 1 4 | -225 | -0.58 | -0.58 | | 0.64 | 0.64 | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | | ≤ | 0 |
| DM 130 | -102 | -0.29 | -0.29 | | 0.32 | 0.32 | | | | | | | | | | ≤ | 0 |
| DM 131 | -407 | -1.16 | -1.16 | | 1.29 | 1.29 | | | | | | | | | | ≤ | 0 |
| DM 132 | -384 | -1.16 | -1.16 | | 1.29 | 1.29 | | | | | | | | | | ≤ | 0 |
| DM 133 | -384 | -1.16 | -1.16 | | 1.29 | 1.29 | | | | | | | | | | ⋮ | |
| ME 1 1 | 4715 | 174 | | | -154 | | | | | | | $\boxed{-8.5}$ | | | | = | 0 |
| ME 1 2 | 5390 | 60 | 174 | | - 67 | -154 | | | | | | | $\boxed{-8.5}$ | | | = | 0 |
| ME 1 3 | 5283 | 60 | 60 | | - 67 | - 67 | | | | | | | | $\boxed{-8.5}$ | | = | 0 |
| ME 1 4 | 2544 | 30 | 30 | | - 33 | - 33 | | | | | | | | | | = | 0 |
| ⋮ | | | | | | | | | | | | | | | | ⋮ | |
| ME 130 | 841 | 15 | 15 | | - 17 | -17 | | | | | | | | | | = | 0 |
| ME 131 | 3393 | 60 | 60 | | - 67 | -67 | | | | | | | | | | = | 0 |
| ME 132 | 2385 | 60 | 60 | | - 67 | -67 | | | | | | | | | | = | 0 |
| ME 133 | 2516 | 60 | 60 | | - 67 | | | | | | | | | | | = | 0 |
| MY 1 1 | -742 | - 7.2 | | | 8.1 | 8.1 | | 1 | | | | | | | | = | 0 |
| MY 1 2 | -777 | -11.6 | -7.2 | | 12.9 | 12.9 | | | 1 | | | | | | | = | 0 |
| MY 1 3 | -721 | -11.6 | -11.6 | | 12.9 | 12.9 | | | | 1 | | | | | | = | 0 |
| ⋮ | | | | | | | | | | | | | | | | ⋮ | |
| CPR 1 1 | | | | | | | | | | | | -0.5 | | | | ≤ | 0 |
| CPR 1 2 | | | | | | | | | | | | | -0.5 | | | ≤ | 0 |
| CPR 1 3 | | | | | | | | | | | | | | -0.5 | | ≤ | 0 |
| CPR 1 4 | | | | | | | | | | | | | | | | ≤ | 0 |
| CPR 1 5 | | | | | | | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | | ⋮ | |

| | COW 1 | $\Delta w^+$ 1 1 | $\Delta w^+$ 1 2 | … | $\Delta w^-$ 1 1 | $\Delta w^-$ 1 2 | … | Milk 1 1 | Milk 1 2 | Milk 1 3 | … | GSlg 1 1 1 | GSlg 1 2 1 | GSlg 1 3 1 | … | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MzPR 1 1 | | | | | | | | | | | | -0.3 | | | | ≤ | 0 |
| MzPR 1 2 | | | | | | | | | | | | | -0.3 | | | ≤ | 0 |
| MzPR 1 3 | | | | | | | | | | | | | | -0.3 | | ≤ | 0 |
| MzPR 1 4 | | | | | | | | | | | | | | | | ≤ | 0 |
| MzPR 1 5 | | | | | | | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | | ⋮ | |
| DMin1 | | | | | | | | | | | | | | | | = | 0 |
| DMin2 | | | | | | | | | | | | | | | | = | 0 |
| DMout1 | | | | | | | | | | | | 1 | 1 | 1 | | ≤ | 0 |
| DMout2 | | | | | | | | | | | | | | | | ≤ | 0 |
| MEin1 | | | | | | | | | | | | | | | | = | 0 |
| MEin2 | | | | | | | | | | | | | | | | = | 0 |
| MzBank | | | | | | | | | | | | | | | | ≤ | 0 |
| MxLoss1 | -35 | | | | 1 | 1 | | | | | | | | | | ≤ | 0 |
| LWbal1 | 24 | -1 | -1 | | 1 | 1 | | | | | | | | | | = | 0 |
| MxCh 1 1 | -14 | 1 | | | 1 | | | | | | | | | | | ≤ | 0 |
| MxCh 1 2 | -14 | | 1 | | | 1 | | | | | | | | | | ≤ | 0 |
| MxCh 1 3 | -14 | | | | | | | | | | | | | | | ≤ | 0 |
| MxCh 1 4 | -7 | | | | | | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | | ⋮ | |
| Quota | | | | | | | | 1 | 1 | 1 | | | | | | ≤ | 875 000 |
| NetMrgn | -90 | | | | | | | 0.19 | 0.19 | 0.19 | | | | | | = | z |

| | GSlg 1 1 2 | GSlg 1 2 2 | GSlg 1 3 2 | … | Conc 1 1 | Conc 1 2 | Conc 1 3 | … | MachSet | DMSilo1 | DMSilo2 | MESilo1 | MESilo2 | … | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Area | | | | | | | | | | | | | | | = | A |
| Land 5 | | | | | | | | | | | | | | | = | 0 |
| Land 6 | | | | | | | | | | | | | | | = | 0 |
| Land 7 | | | | | | | | | | | | | | | = | 0 |
| Land 8 | | | | | | | | | | | | | | | = | 0 |
| Land 9 | | | | | | | | | | | | | | | = | 0 |
| Land10 | | | | | | | | | | | | | | | = | 0 |
| Land11 | | | | | | | | | | | | | | | = | 0 |
| ⋮ | | | | | | | | | | | | | | | ⋮ | |
| HrvTime1 | | | | | | | | | -24 | | | | | | ≤ | 0 |
| HrvTime2 | | | | | | | | | -24 | | | | | | ≤ | 0 |
| DM 1 1 | 1 | | | | 0.84 | | | | | | | | | | ≤ | 0 |
| DM 1 2 | | 1 | | | | 0.84 | | | | | | | | | ≤ | 0 |
| DM 1 3 | | | 1 | | | | 0.84 | | | | | | | | ≤ | 0 |
| DM 1 4 | | | | | | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | | |
| DM 130 | | | | | | | | | | | | | | | | |
| DM 131 | | | | | | | | | | | | | | | | |
| DM 132 | | | | | | | | | | | | | | | | |
| DM 133 | | | | | | | | | | | | | | | ⋮ | |
| ME 1 1 | **-8.5** | | | | -12.8 | | | | | | | | | | = | 0 |
| ME 1 2 | | **-8.5** | | | | -12.8 | | | | | | | | | = | 0 |
| ME 1 3 | | | **-8.5** | | | | -12.8 | | | | | | | | = | 0 |
| ME 1 4 | | | | | | | | | | | | | | | = | 0 |
| ⋮ | | | | | | | | | | | | | | | ⋮ | |
| ME 130 | | | | | | | | | | | | | | | = | 0 |
| ME 131 | | | | | | | | | | | | | | | = | 0 |
| ME 132 | | | | | | | | | | | | | | | = | 0 |
| ME 133 | | | | | | | | | | | | | | | = | 0 |
| MY 1 1 | | | | | | | | | | | | | | | = | 0 |
| MY 1 2 | | | | | | | | | | | | | | | = | 0 |
| MY 1 3 | | | | | | | | | | | | | | | = | 0 |
| ⋮ | | | | | | | | | | | | | | | ⋮ | |
| CPR 1 1 | -0.5 | | | | 0.5 | | | | | | | | | | ≤ | 0 |
| CPR 1 2 | | -0.5 | | | | 0.5 | | | | | | | | | ≤ | 0 |
| CPR 1 3 | | | -0.5 | | | | 0.5 | | | | | | | | ≤ | 0 |
| CPR 1 4 | | | | | | | | | | | | | | | ≤ | 0 |
| CPR 1 5 | | | | | | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | ⋮ | |

| | GSlg 1 1 2 | GSlg 1 2 2 | GSlg 1 3 2 | ... | Conc 1 1 | Conc 1 2 | Conc 1 3 | ... | MachSet | DMSilo1 | DMSilo2 | MESilo1 | MESilo2 | ... | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| MzPR 1 1 | -0.3 | | | | -0.3 | | | | | | | | | | ≤ | 0 |
| MzPR 1 2 | | -0.3 | | | | -0.3 | | | | | | | | | ≤ | 0 |
| MzPR 1 3 | | | -0.3 | | | | -0.3 | | | | | | | | ≤ | 0 |
| MzPR 1 4 | | | | | | | | | | | | | | | ≤ | 0 |
| MzPR 1 5 | | | | | | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | ⋮ | |
| DMin1 | | | | | | | | | | -1 | | | | | = | 0 |
| DMin2 | | | | | | | | | | | -1 | | | | = | 0 |
| DMout1 | 1 | 1 | 1 | | | | | | | -1 | | | | | ≤ | 0 |
| DMout2 | | | | | | | | | | | -1 | | | | ≤ | 0 |
| MEin1 | | | | | | | | | | | | -1 | | | = | 0 |
| MEin2 | | | | | | | | | | | | | -1 | | = | 0 |
| MzBank | | | | | | | | | | | | | | | ≤ | 0 |
| MxLoss1 | | | | | | | | | | | | | | | ≤ | 0 |
| LWbal1 | | | | | | | | | | | | | | | = | 0 |
| MxCh 1 1 | | | | | | | | | | | | | | | ≤ | 0 |
| MxCh 1 2 | | | | | | | | | | | | | | | ≤ | 0 |
| MxCh 1 3 | | | | | | | | | | | | | | | ≤ | 0 |
| MxCh 1 4 | | | | | | | | | | | | | | | ≤ | 0 |
| ⋮ | | | | | | | | | | | | | | | ⋮ | 0 |
| Quota | | | | | | | | | | | | | | | ≤ | Quota |
| NetMrgn | | | | | -155 | -155 | -155 | | -16700 | | | | | | = | z |

The Metabolizable Energy (ME) content of both silages are initially 8.5 MJ/kg DM. The LP is solved and the optimal values of DMSilo1, DMSilo2, MESilo1 and MESilo2 are determined. The ME content of silages 1 and 2 are then updated according to the equations (5.16) and (5.17).

$$MEs_1 = \frac{MESilo_1}{DMSilo_1} \tag{5.16}$$

$$MEs_2 = \frac{MESilo_2}{DMSilo_2} \tag{5.17}$$

The LP model is then rerun and the process is repeated until a convergence occurs.

# Chapter Six

## 6. Computer programme: main features

A computer programme has been developed and gathers all the necessary technical and economic data, allows data to be changed interactively and generates the linear programming (LP) matrix. It was written in Basic with the software MS-QuickBasic PDS 7.1 (Microsoft QuickBasic Professional Developer System version 7.1).

The matrix generator programme creates a file with the LP matrix in a standard MPS format (See Appendix V for details of the standard MPS format). It was developed in such a way that users need no knowledge of LP or computer programming. Users are only expected to input data such as which months cows are allowed to calve, machinery available for silage-making, milk yield and milk price (and seasonal adjustments), etc.

The commercial LP solver XPRESS-MP was used to solve the LP, although any other LP solver capable of reading an LP matrix in a standard MPS format can be used.

In this chapter the following topics will be presented and discussed:

- division of the year in two groups of periods: $S_1$ and $S_2$
- recursion approach to determine grass silage digestibility
- inclusion of a constraint specifying a minimum number of cows
- conversion of the seasonality of milk prices from monthly to the periods of $S_2$
- standard liveweight change assumed by the LP model
- data provided by the programme
- report writer programme and interpretation of the mathematical solution

## 6.1. Periods of the year for the LP model

The year is divided into two sets of periods, in order to adequately relate grass growth (for grazing and for silage), nutritional requirements, milk production and feeding strategy throughout the year. In one set ($S_1$), the year was divided into thirteen 4-week periods. The periods of this set indicate the calving periods and the periods of the beginning the lactation. The objective of this set was to have the year divided into periods with same number of days rather than twelve months with different numbers of days. Table 6.1 shows the thirteen periods with their dates and weeks of the year.

**Table 6.1 - Thirteen 4-week periods of the year ($S_1$)**

| Period | Week of the year | Date |
|---|---|---|
| 1 | 1- 2- 3- 4 | 04 Jan - 31 Jan |
| 2 | 5- 6- 7- 8 | 01 Feb - 28 Feb |
| 3 | 9-10-11-12 | 01 Mar - 28 Mar |
| 4 | 13-14-15-16 | 29 Mar - 25 Apr |
| 5 | 17-18-19-20 | 26 Apr - 23 May |
| 6 | 21-22-23-24 | 24 May - 20 Jun |
| 7 | 25-26-27-28 | 21 Jun - 18 Jul |
| 8 | 29-30-31-32 | 19 Jul - 15 Aug |
| 9 | 33-34-35-36 | 16 Aug - 12 Sep |
| 10 | 37-38-39-40 | 13 Sep - 10 Oct |
| 11 | 41-42-43-44 | 11 Oct - 07 Nov |
| 12 | 45-46-47-48 | 08 Nov - 05 Dec |
| 13 | 49-50-51-52 | 06 Dec - 03 Jan |

In the other set (S2), the year was divided into thirty-three periods. During the grass growth season, periods last 1 week and during the rest of the year, they last 4 weeks. There is only one exception: the period immediately before the beginning of the grass growth season — period 4 — which lasts 2 weeks. Periods in this set are used for grass land use and schedule, milk production, nutrient requirements, liveweight changes and also for the periods when cows are allowed to eat silage and concentrates. Table 6.2 shows the thirty-three periods with their dates and weeks of the year. It is assumed that the period for utilisation of grass starts at the middle of the 15th week (102th day of the year — 12 April).

**Table 6.2 - Thirty-three periods of the year ($S_2$)**

| Period | Week of the year | Date |
|---|---|---|
| 1 | 1- 2- 3- 4 | 04 Jan - 31 Jan |
| 2 | 5- 6- 7- 8 | 01 Feb - 28 Feb |
| 3 | 9-10-11-12 | 01 Mar - 28 Mar |
| 4 | 13-14 | 29 Mar - 11 Apr |
| 5 | 15 | 12 Apr - 18 Apr |
| 6 | 16 | 19 Apr - 25 Apr |
| 7 | 17 | 26 Apr - 02 May |
| 8 | 18 | 03 May - 09 May |
| 9 | 19 | 10 May - 16 May |
| 10 | 20 | 17 May - 23 May |
| 11 | 21 | 24 May - 30 May |
| 12 | 22 | 31 May - 06 Jun |
| 13 | 23 | 07 Jun - 13 Jun |
| 14 | 24 | 14 Jun - 20 Jun |
| 15 | 25 | 21 Jun - 27 Jun |
| 16 | 26 | 28 Jun - 04 Jul |
| 17 | 27 | 05 Jul - 11 Jul |
| 18 | 28 | 12 Jul - 18 Jul |
| 19 | 29 | 19 Jul - 25 Jul |
| 20 | 30 | 26 Jul - 01 Aug |
| 21 | 31 | 02 Aug - 08 Aug |
| 22 | 32 | 09 Aug - 15 Aug |
| 23 | 33 | 16 Aug - 22 Aug |

Table 6.2. Continued

| Period | Week of the year | Date |
|---|---|---|
| 24 | 34 | 23 Aug - 29 Aug |
| 25 | 35 | 30 Aug - 05 Sep |
| 26 | 36 | 06 Sep - 12 Sep |
| 27 | 37 | 13 Sep - 19 Sep |
| 28 | 38 | 20 Sep - 26 Sep |
| 29 | 39 | 27 Sep - 03 Oct |
| 30 | 40 | 04 Oct - 10 Oct |
| 31 | 41-42-43-44 | 11 Oct - 07 Nov |
| 32 | 45-46-47-48 | 08 Nov - 05 Dec |
| 33 | 49-50-51-52 | 06 Dec - 03 Jan |

## 6.2. Recursion approach to determine the grass silage digestibility

Recursion is the approach adopted to determine the grass silage digestibility. This technique, also known as Successive Linear Programming (SLP), deals with some type of non-linear programming problems. The determination of silage digestibility is a typical problem to be solved by this technique.

The non-linearity of the problem was shown in equation (4.6). Equation (6.1) is a simplified version of equation (4.6) (Energy balance constraints), showing only those terms concerning grass silage and energy requirements.

$$\cdots \quad + MEs_q . s_{ijq} = ME_{req} \tag{6.1}$$

where $ME_{req}$ is the ME required for maintenance, milk production and weight gain and

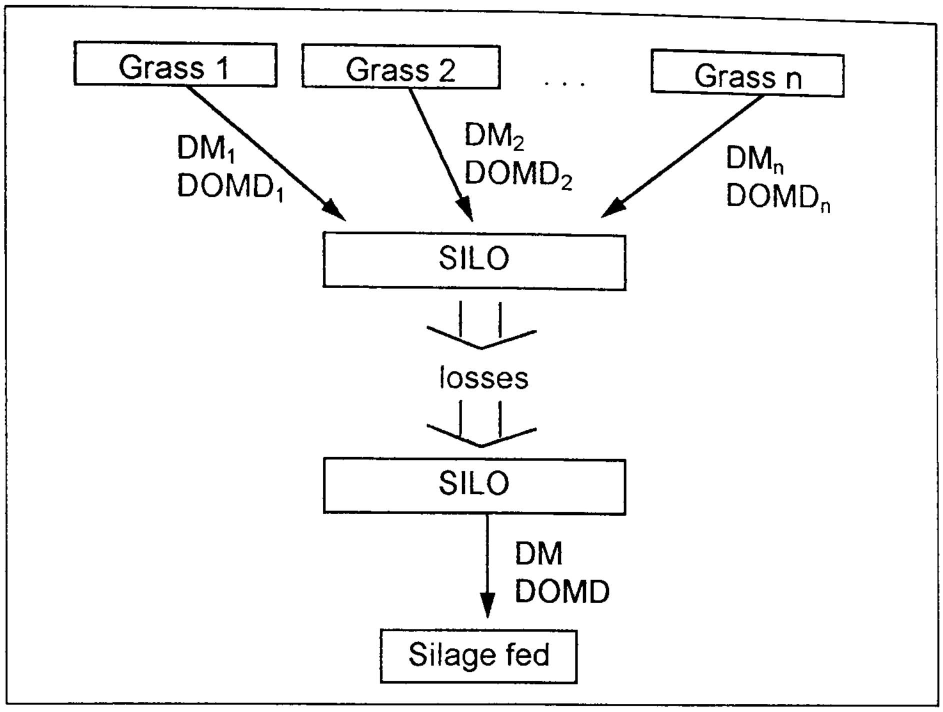
$MEs_q$ is the ME content of grass silage quality (q).

$MEs_q$ is calculated according to equation (6.2)

$$MEs_q = \frac{q_{ME}}{q_{DM}} = \frac{\sum_k MEh_{jk}.(1 - DMs_{loss}).DMh_{jk}.h_{jk}}{\sum_k (1 - DMs_{loss}).DMh_{jk}.h_{jk}} \tag{6.2}$$

The quality of the grass silage is also dependent on the conservation system that is adopted (losses). If silage is made of grass cut only at a specific time, and consequently with a specific digestibility, it is easy to determine its digestibility and energy content. The problem arises when grass fields are cut at different dates and ensiled together. The grass from each field, with different digestibility, is ensiled into a silo. The digestibility of this "mix of grasses" is determined by the proportion of each one in the silo. Figure 6.1 illustrates the problem.

**Figure 6.1 - Silage made with grass cut in different periods**

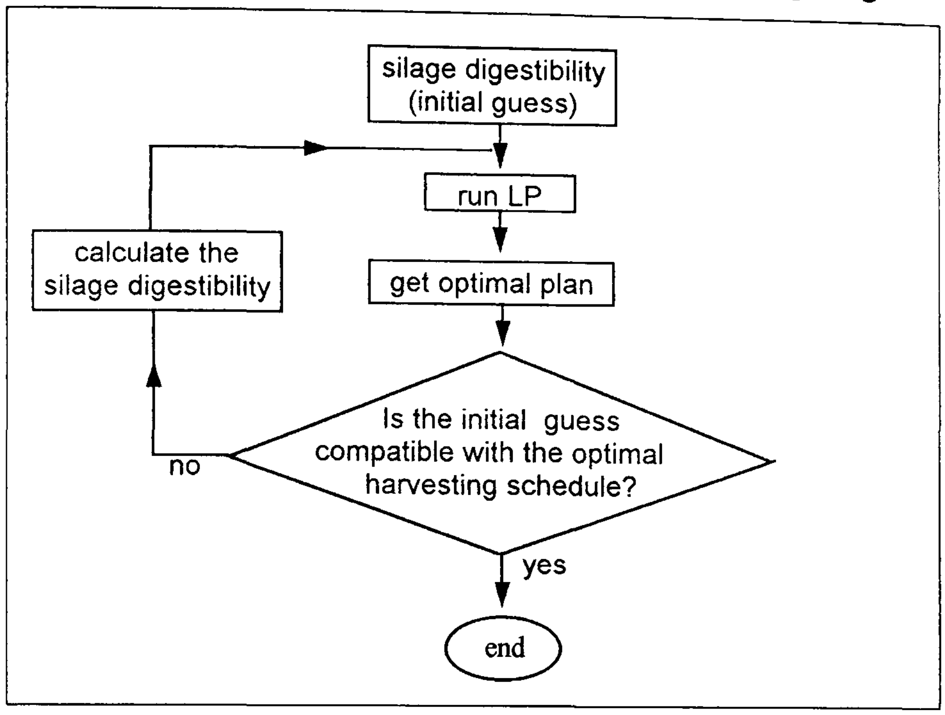



The amount of each type of grass that should be harvested and ensiled is determined by the linear programme (LP). Since the digestibility of the grass ensiled is not known in advance, the digestibility of the silage that will be fed is not known.

Simultaneously, the LP determines the optimal herd size, calving pattern and feeding strategy. In order to achieve this, the amount of grass silage that will be available, as well as its digestibility, must be known in advance.

By using the recursion, we can "guess" some reasonable initial values for the digestibility of the silage in each silo and solve the LP with those values. From the optimal solution we check if the initial "guess" is compatible with the grass harvested according to the optimal plan. If this is the case, the initial "guess" was correct. Otherwise, it is necessary to calculate the grass digestibility from the optimal solution of the LP and run the LP again, starting from the last optimal solution. The recursion process continues until convergence occurs (see equations (4.6), (4.9), (4.10) and (4.11)) . Figure 6.2 illustrates the recursion approach adopted in conjunction with the LP model.

Figure 6.2 - Recursion approach to determine grass silage digestibility




It should be noticed that it is not guaranteed that there will be a convergence to the global optimum. It may happen that the problem converges to a local optimum or the problem may even not converge at all (Dash, 1993).

## 6.3. Minimum number of cows

Included in the LP matrix is a constraint that specifies a minimum number of cows. This constraint helps to accelerate the convergence to the optimal solution of the LP. This minimum number of cows forces an initial feasible solution that includes cows and prevents the LP solver from investigating non-optimal solutions that do not include cows.

According to the model, a solution with no cows (but allocating areas for grazing and silage) would be feasible. The mathematical model determines that if there are cows calving in period (i), they must be supplied with enough energy to satisfy their requirements. This energy comes from the feed sources available. However, the opposite is not necessarily true: it is possible to provide energy even if there are no cows in the optimal plan. Equation (6.3) illustrates the problem:

$$c_i \,.\, ME_{ij} \le E_{fed_i} \qquad \forall i \in S_1, \forall j \in S_2 \qquad \textbf{(6.3)}$$

There is a cost associated with the supply of energy and, obviously, the optimal solution will not contain a solution where energy is supplied if there are no cows. The LP solver may take some iterations to include cows spontaneously in the optimal plan. It would be a waste of time to calculate those solutions that will certainly not be optimal. The inclusion of a constraint forcing a minimum number of cows helps the optimal solution to be found faster in many cases. In some cases, the introduction of that constraint does not alter the speed of convergence, as the inclusion of cows in the solution would be done spontaneously in an early stage anyway.

When the optimal solution is found, if the total number of cows is the minimum, this is probably due to that constraint forcing a minimum number of cows. The actual optimal plan for that farm would not include cows at all, which can be verified by removing the constraint.

## 6.4. Seasonal milk prices

The price paid for milk is affected by its composition (fat and protein content) and seasonality. A basic price is calculated according to the fat and protein content of the milk, which is strongly related to breed, and the price paid for these components (p/litre/1% fat and p/litre/1% protein). Final milk prices are calculated by taking into account the seasonal adjustments.

With the year divided into 33 periods (weekly during the grass growth season), it is necessary to convert the 12 monthly price adjustments to the division adopted in the model.

Table 6.3 shows the 52 weeks of the year and the month related to each one. The milk price can be calculated for any period of $S_2$ (Table 6.1), based on the relationship presented in Table 6.3.

**Table 6.3 - Relationship between weeks and months of the year**

| Week number | Month |
|---|---|
| 1 - 2 - 3 - 4 | January |
| 5 - 6 - 7 - 8 | February |
| 9 - 10 - 11 - 12 - 13 | March |
| 14 - 15 - 16 - 17 | April |
| 18 - 19 - 20 - 21 - 22 | May |
| 23 - 24 - 25 - 26 | June |
| 27 - 28 - 29 - 30 | July |
| 31 - 32 - 33 - 34 - 35 | August |
| 36 - 37 - 38 - 39 | September |
| 40 - 41 - 42 - 43 | October |
| 44 - 45 - 46 - 47 - 48 | November |
| 49 - 50 - 51 - 52 | December |

For example, the milk price during period 4 (weeks 13 and 14) is approximated as

$$\frac{\text{March price} + \text{April price}}{2}$$

because week 13 is in March and week 14 is in April. Similarly, the milk price at period 31 (weeks 41, 42, 43 and 44) is estimated to be

$$\frac{3 \times \text{October price} + \text{November price}}{4}$$

because weeks 41, 42 and 43 are in October and week 44 is in November.

## 6.5. Standard liveweight change pattern assumed by the model

A standard liveweight (LW) change pattern was included in the LP model, following the recommendation by MAFF (MAFF, 1984), as previously discussed (Section 3.3.4.1).

It was assumed that the standard LW change pattern would allow the cow over the whole lactation period to regain the weight lost during early lactation. The standard LW change pattern of the model is based on the average weight change recommended by MAFF (1984): an average loss of 0.5 kg/day during the first 10 weeks of lactation followed by a 10-week period of no weight change. In order to produce a progressive liveweight loss, it was assumed that the loss is higher during the first ten weeks of lactation ( 1 kg/day during the first 3 weeks), and lower during the subsequent weeks ( 0.4 kg/day for the next 3 weeks, and 0.2 kg/day for the next 4 weeks). This makes a total loss of 35 kg at the end of a period of ten weeks (average of 0.5 kg/day).

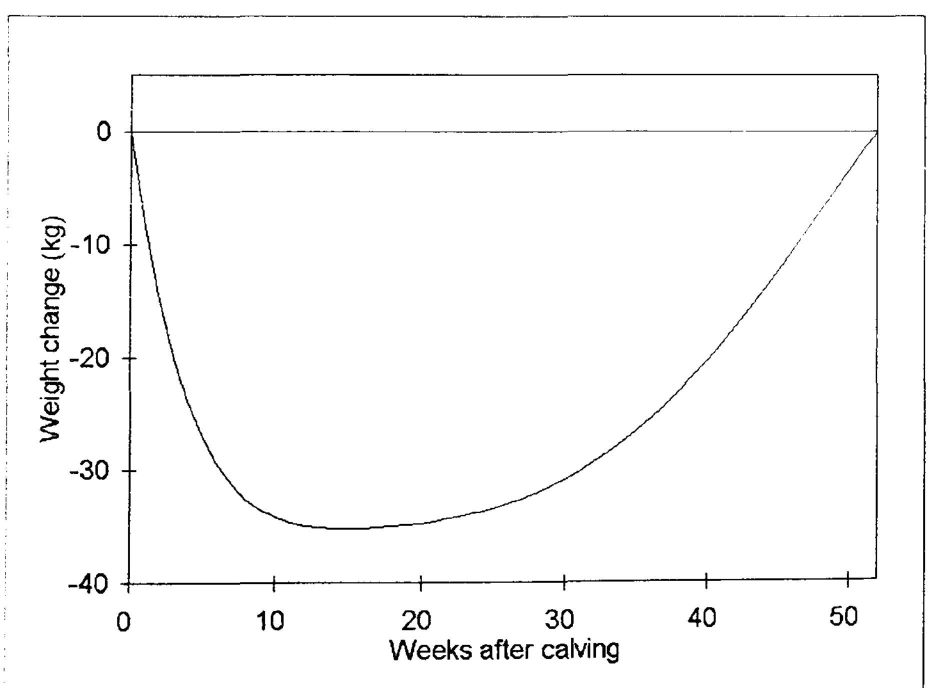
The standard weight gain is smaller than the one followed by MAFF because we assumed mature cows (final weight aimed to be the same as the initial weight), while MAFF assumes growth.

Table 6.4 summarizes and Figure 6.3 illustrates the standard weight change throughout the year.

**Table 6.4 - Standard liveweight changes throughout the year**

| Weeks after calving | Liveweight change (kg/day) |
|---|---|
| 1 - 3 | -1.000 |
| 4 - 6 | -0.400 |
| 7 - 10 | -0.200 |
| 11 - 20 | 0.000 |
| 21 - 30 | 0.050 |
| 31 - 40 | 0.150 |
| 41 - 52 | 0.250 |

**Figure 6.3 - Standard cumulative liveweight change throughout the year**




It should be remembered that the decision variables of the LP model concerning weight loss or weight gain represent a loss or gain in addition to those weight changes already considered in the model.

## 6.6. Data provided

The computer programme provides all necessary data to run the model without any user intervention to change or add data. Users can change all items concerning economic data and most items concerning technical data, although this is unlikely to be necessary. In general, users will change some few items of data to adjust the model to their particular farm conditions (e.g., farm size, milk quota, milk prices, machinery available, etc.).

Each item of the data provided by the programme and technical coefficients used by the programme are described below.

### 6.6.1. Herd structure

Table 6.5 shows the herd structure used in the model, based on the herd structure used by Olney and Falconer (1985).

**Table 6.5 - Herd structure**

| Lactation | % of herd |
|---|---|
| 1 | 20 |
| 2 | 15 |
| 3 | 13 |
| ≥ 4 | 52 |

### 6.6.2. Parameters to estimate milk yield

Table 6.6 shows the technical parameters assumed by the model to describe the lactation curves for each lactation and the herd structure. Parameters (B) and (C) were extracted from Wood (1969).

**Table 6.6 - Curve shape parameters**

| Lactation | A | B | C |
|---|---|---|---|
| 1 | 0.9126 | 0.15 | 0.03 |
| 2 | 0.8709 | 0.21 | 0.04 |
| 3 | 0.9257 | 0.20 | 0.05 |
| ≥ 4 | 0.8724 | 0.24 | 0.05 |

The averages for the parameters (A), (B) and (C), according to the herd structure, are as follows:

A = 0.8874 B = 0.2123 C = 0.0445

### 6.6.3. Milk yield composition

The programme assumes that all cows of the herd have a milk composition as previously shown in Table 5.6 (Friesian):

- Fat content: 39.8 g/kg ( 3.98 % )
- Solids non-fat: 86 g/kg ( 8.6 % )

### 6.6.4. Dry matter intake (DMI)

Table 6.7 presents the initial liveweight (LW) of cows in each lactation, the herd structure and the average LW of cows in the herd. It is assumed that the dry matter intake will not vary with weight changes, as previously discussed in section 5.2. DMI is calculated with the initial LW of the average cow of the herd. Furthermore, cows are fed to achieve the final LW.

**Table 6.7 - Standard milk yield, initial and final liveweight of cows in each lactation**

| Lactation | Initial LW | Final LW | Standard milk yield |
|---|---|---|---|
| 1 | 450 | 500 | 4542 |
| 2 | 500 | 550 | 5044 |
| 3 | 550 | 600 | 5471 |
| $\geq 4$ | 600 | 600 | 6085 |

The averages for the standard milk yield, initial and final liveweights, according to the herd structure, are as follows:

| | |
|---|---|
| Standard milk yield | : 5578 kg/year |
| Initial LW | : 548.50 kg |
| Final LW | : 572.50 kg |

### 6.6.5. Parameters to estimate energy requirements

Energy requirements are calculated according to the models provided by MAFF (1984).

### 6.6.6. Grass yield and digestibility

This LP model does not consider the fertilizer application as a decision strategy and consequently does not optimize it. However, grass yield is based on a determined rate of fertilizer application. The matrix generator reads two files that provide the dry matter (DM) yield and D-value (%) of grass at fields for grazing and for silage-making with an annual application of 350 kg N/ha. This data is the original data from experiments at the Grassland Research Institute (GRI) and was also used by Audsley (1984). Dry matter yield and D-values (%) are shown in Appendix IV.

The LP model reads two files with data of grass yield and digestibility from fields for grazing and fields for silage-making. The data can be changed by replacing or editing these files.

A grazing efficiency of 60% is assumed, which means there is a loss of 40% of DM during grazing. It is assumed there is no loss concerning digestibility.

### 6.6.7. Silage losses: dry matter (DM) and digestible organic matter (D-value) losses

Data provided concerning dry matter losses of grass silage assumes that grass is ensiled with 25% DM (after wilting). It is also assumed that a precision chop forage harvester is used and that silage will contain additive, applied at a standard rate. These figures are in agreement with losses presented by McGechan (1989), who summarized results of several experiments by several authors.

Table 6.8 shows data concerning DM losses and estimates of proportion of digestible organic matter lost (D-value) as provided by the programme.

**Table 6.8 - Dry matter losses and estimates of proportion of digestible component lost**

| | DM loss % | % of digestible nutrients | D-value loss % |
|---|---|---|---|
| **Field losses** | | | |
| respiration | 4.8 | 100 | 4.8 |
| leaching by rain | 1.0 | 100 | 1.0 |
| mechanical treatment | 1.5 | 50 | 0.75 |
| **In-silo losses** | | | |
| air-infiltration during filling | 1.0 | 100 | 1.0 |
| air-infiltration during feed-out (includes surface waste) | 8.0 | 100 | 8.0 |
| invisible losses | 7.0 | 50 | 3.5 |
| effluent | 0.5 | 50 | 0.25 |

Total DM loss and total D-value are calculated as 23.8 % and 19.3 %, respectively.

### 6.6.8. Field operations: machinery, system and efficiency

Although the model can use any system of silage making specified by the workrate for the operation, for the purposes of the matrix generator programme only one system has been incorporated as follows:

- wilted silage will be made so the mowing operation will be performed separately
- density of grass ensiled: 150 kgDM/m$^3$ (estimated from McGechan, 1990a; for silage with 25% DM and 65% D-value)
- 3-man system: one man on the tractor with the forage harvester, a second man transporting a second trailer from the field to the clamp and vice-versa, and a third man on the tractor filling the clamp
- three-in-line system: a tractor pushing the forage harvester and pulling a trailer
- silage must be made in no more than 24 hours

The workrate is then calculated using the formulae (5.14) and (5.15). All the parameters in these equations can changed by the user. The programme assumes that the most powerful tractor will be used for harvesting with the second most powerful tractor being used for mowing. It should be noticed that when wilted silage is made, the model assumes that the tractor used for mowing is the same tractor used to fill the clamp

Table 6.9 presents the combination of machines that the programme provides to make silage.

**Table 6.9 - Machinery to make silage: tractors and implements for each operation**

| Operation | Implement | Tractor |
|---|---|---|
| mowing | 2.1 m drum mower-conditioner | 65 kW (4 WD) |
| harvesting | precision chop forage harvester + pick-up device 2.1 m wide + 6 t silage trailer | 75 kW ( 4 WD ) |
| transporting | second silage trailer (6 t) | 45 kW ( 2 WD ) |
| filling clamp | buckrake | 65 kW ( 4 WD ) |

## 6.6.9. Data to calculate harvester's forward speed and workrate to make silage

The data the programme provides to calculate forage harvester's forward speed according to equations (5.14) and (5.15) is shown below:

- weight of tractor plus harvester: 15.2 t (three-in-line system with a full 6 t trailer)
- maximum forward speed: 7.2 km/h
- chop length: 30 mm
- slope: 0º
- trailer: 2 x 6 t trailers — Approx. volume of trailers
- density: 150 kg/m$^3$ — 2 x 10 m$^3$ = 20 m$^3$
- field efficiency: 66 % (three-in-line system)
- trailer change time: 2 min.
- maximum throughput capacity: 40 t/h

We assume that mowing operation is performed separately (wilted silage) and the programme provides the following figures concerning this operation:

- pick-up width: 2.1 m
- work rate: 2 ha/h

## 6.6.10. Data for concentrates

Table 6.10 shows the values which the programme provides for maximum proportion of concentrates in the ration, reduction due to "substitution effect", metabolizable energy (ME) content and percentage of dry matter (%DM).

**Table 6.10 - Data for concentrates: maximum proportion, substitution effect, ME and %DM content**

| Maximum proportion (%) | Substitution effect (kg/kg) | ME (MJ/kgDM) | (%DM) |
|---|---|---|---|
| 50 | 0.84 | 12.8 | 86 |

### 6.6.11. Data for maize silage

Data concerning maize crop fresh yield, dry matter content, dry matter loss of silage, D-value and Metabolisable Energy provided by the programme are shown in Table 6.11.

**Table 6.11 - Data for maize crop and maize silage**

| | |
|---|---|
| Yield (t/ha) | 40 |
| %DM | 33 |
| %DM loss | 20 |
| D-value (%) | 70 |
| ME (MJ/kgDM) | 11.2 |
| Maximum proportion (%) | 30 |

## 6.7. The Report Writer programme

A report writer programme was developed to present the solution of the LP in a more user orientated way. The solution file generated by the LP solver contains the values of the decision variables. These values are of no use to the normal user unless they are interpreted. For example, it would be difficult for everyone to understand what **G 415 3** = **10** or **MZ 5 3** = **2** meant. The report writer extracts the solution from the file created by the LP solver, interprets that solution and generates a file more easily understandable. For example, **G 415 3** = **10** in the optimal plan means that cows that calved in period 4 should graze 10 hectares at period 15, in a field where grass was allowed to grow for 3 weeks. **MZ 5 3** = **2** means that cows that calved in period 5 should be fed 2 t of maize silage during period 3.

In studying these reports one should be aware that apparently spurious effects can be generated due to averaging over periods which are not the same as the periods used in the model. For example, one of the problems that occurs when a report summarizes the results is concerned with averages from periods with different lengths. The report may contain some values that appear to behave strangely. For example, let us suppose that the optimal solution has been found. It contains the following information: cows calving in period (x) should be fed 1 kg of concentrates per day, from week 9 to week 14. Table 6.1 shows that weeks 9 to 12 are in period 3, and weeks 13 and 14 are in period 4. Although the daily average consumption of concentrates during these six weeks is 1 kg/day, a report based on 4-week periods will present an average consumption of 1 kg/day during period 3 and 0.5 kg/day during period 4. Similar effects may occur with consumption of any other source of energy as well as with milk yield.

## 6.8. Summary

The diagram below illustrates the sequence of operations performed by the computer programs (Matrix Generator and Report Writer) as well as the LP solver (XPRESS-MP) in order to obtain the solution of a problem.

**Figure 6.4 - Flow chart**

**Matrix Generator**

- Estimates economic and technical data
- Read files GRASS1.DAT, GRASS2.DAT and WKSTRU.DAT to input data for grass growth and division of periods
- Creates auxiliar files .AX1 and .AX2
- Generates the LP matrix (MPS format) (.MAT)

↓

**XPRESS-MP**

- Solves LP
- Creates the solution file .SOL (binary)

↓

**Report Writer**

- Reads the auxiliar files .AX1 and .AX2 and the solution file .SOL
- Generates the report file .TXT (ASCII)

# Chapter Seven

## 7. Discussion of results

The purpose of this chapter is to present the results of different scenarios and discuss how optimal plans are affected by different farm situations and conditions.

The main effects are summarized under the following topics:

- net margin
- marginal price of milk quota
- average annual silage and consumption of concentrates
- calving pattern
- total milk production
- liveweight change throughout the lactation

It is assumed that cash crops can be grown on the farm as an alternative plan and the value of these represents the opportunity cost of land. It is also assumed that all operations related to any cash crop will be performed by contractors.

### 7.1. Description of the two standard systems

Before the analysis of the results is explained a description of the two standard dairy farm systems is given below. Prices were extracted from *Farmers Weekly* May/95 and Nix (1995), where average prices were estimated for the 1995 calendar year, based on figures of previous years.

*Standard System 1: (Scenario A1)*

- Area: 70 ha
- Milk quota: 850 000 litres
- Opportunity cost of land (for cash crop): £ 400 /ha
- Milk
    - . composition: 3.89 % FAT and 3.21 % Protein
    - . compositional prices: 2.292 p/1% FAT and 3.580 p/1% protein
    - . cost of transport: 0.12 p/litre
- Seasonal adjustments of milk prices
    - . Jan-Mar and Nov-Dec: – 0.70 p/litre
    - . Apr: – 2.00 p/litre
    - . May: – 2.50 p/litre
    - . Jun: – 1.00 p/litre
    - . Jul-Aug: + 3.00 p/litre
    - . Sep: + 2.00 p/litre
    - . Oct: + 1.00 p/litre

- Concentrates
  - price: £ 155 /tDM
  - energy content: 12.8 MJ/kgDM
- Grazing areas
  - Annual cost (seed + fertilizer + sprays): £ 135 /ha
- Grass silage areas:
  - Annual cost (seed + fertilizer + sprays): £ 222 /ha
  - Grass silage can be fed from autumn to the beginning of spring, when grass cannot be harvested and ensiled (i.e. periods 1 to 9 and 31 to 33)
- Cow depreciation:
  - Replacement rate: 20%
  - Cost of replacement (20 months old): £ 1100
  - Cull cow price: £ 525
  - Misc. variable costs (veterinary, artificial insemination, bedding, etc) £ 95
  - Annual value of calves £ 120
- Annual cost of machinery
  - Tractors
    - 75 kW (4WD) £ 4000
    - 65 kW (4WD) £ 3300
    - 45 kW (2WD) £ 2200
  - Forage harvester
    - Precision-chop (40 t/h) £ 2300
  - Mower-conditioner £ 1000
  - Silage trailer (6 t) £ 450
- Fuel price : £ 0.12 / litre
- Annual labour cost: £ 12500
- Specialist machinery: Forage harvester+mower-conditioner+silage trailer

  Annual cost: £ 2300 + £ 1000 + 2 x £ 450 = £ 4200
- Non-specialist machinery: tractors.

  Annual cost converted to hourly cost based on a use of 500 hours a year (Nix, 1995)
- Technical coefficients (e.g., grass growth, machinery set, silage losses, etc) as previously described in Chapter 6

*Standard System 2: (Scenario A3)*

The same as System 1, but with maize crop area up to 10 ha. Maize silage is being fed from January to August, and from November to December.

- Maize silage
  . Annual cost of maize crop: £ 215 /ha
  . Harvesting cost: £ 115 /ha (assumed to be contracted)
  . Yield of maize crop: 40 t/ha (33% DM)
  . Energy content: 11.2 MJ/kgDM
  . DM loss: 20%
  . Maximum proportion in ration: 30%

## 7.2. Effects of maize crop

In these scenarios, maximum maize crop area was increased from 0 to 20 ha with increments of 5 ha. The results are summarized in Table 7.1. Notice that the maximum maize crop area selected by the model as profitable was 16.5 ha.

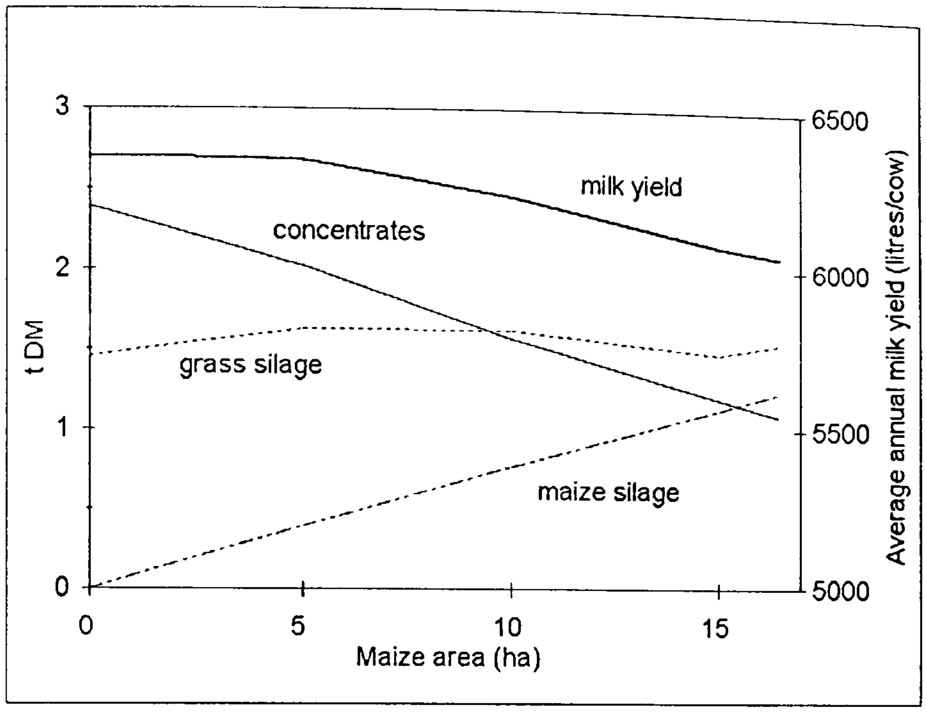
**Table 7.1 - Effects of maize crop: summary of results**

| Maize crop (ha) | Net margin (£) | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| 0.0 | 93 127 | 8.24 | 6351 |
| 5.0 | 97 798 | 8.43 | 6340 |
| 10.0 | 101 846 | 8.68 | 6227 |
| 15.0 | 104 773 | 9.29 | 6077 |
| 16.5 | 105 577 | 9.44 | 6044 |

| Maize crop | Average annual consumption (tDM/cow) | | | Calving Pattern | | |
|---|---|---|---|---|---|---|
| (ha) | Grass Silage | Maize Silage | Concentrates | (Number of cows) | | |
| 0.0 | 1.46 | 0.00 | 2.39 | Feb: 13 | Jun/Jul: 81 | Jul/Aug: 40 |
| 5.0 | 1.63 | 0.39 | 2.02 | Feb: 36 | Jun/Jul: 98 | |
| 10.0 | 1.62 | 0.77 | 1.57 | Feb: 53 | Jun/Jul: 83 | |
| 15.0 | 1.48 | 1.13 | 1.20 | Feb: 59 | Jun/Jul: 80 | |
| 16.5 | 1.54 | 1.24 | 1.09 | Mar: 68 | Jun/Jul: 72 | |

Milk quota was always the limiting factor in these scenarios. When the maize crop area was increased, maize silage replaced concentrates, while grass silage consumption was almost the same and feeding level and hence average milk yield per cow was slightly reduced (Figure 7.1). The rate of consumption of concentrates per unit of milk produced, however, was reduced drastically (from 0.38 to 0.18 kg/litre). Marginal price of milk quota indicates how much would be worth paying for extra quota.

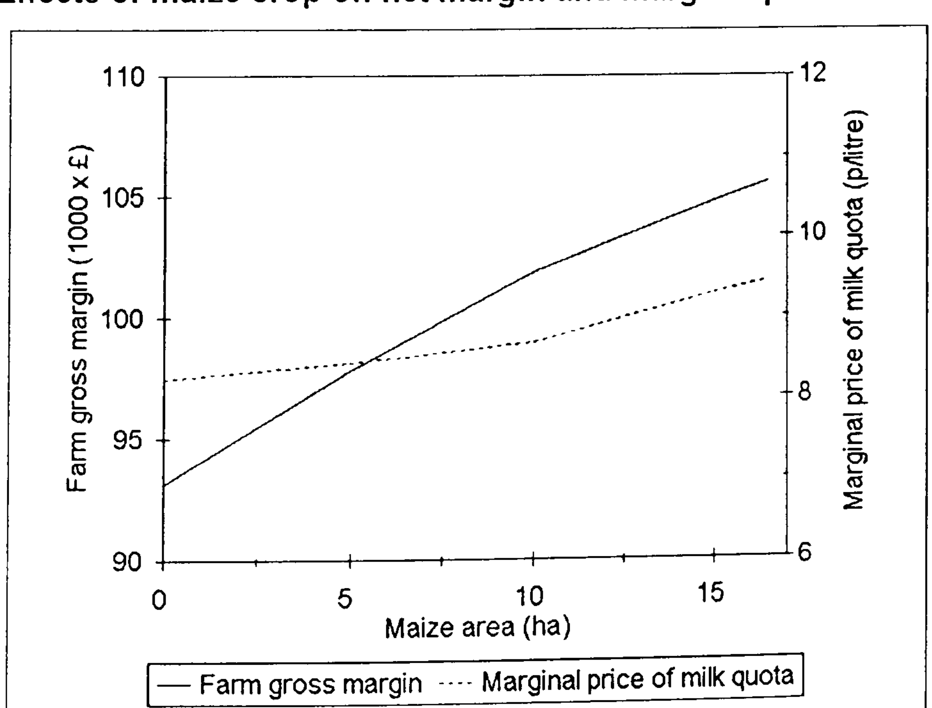
**Figure 7.1 - Effects of maize crop: silage and concentrates consumption and the average annual milk yield**




The net margin increased with maize area, but at lower rates when the maize area was larger. This suggests that there is an optimal maize area according to the farm conditions and growing areas above that optimal will not bring any benefit.

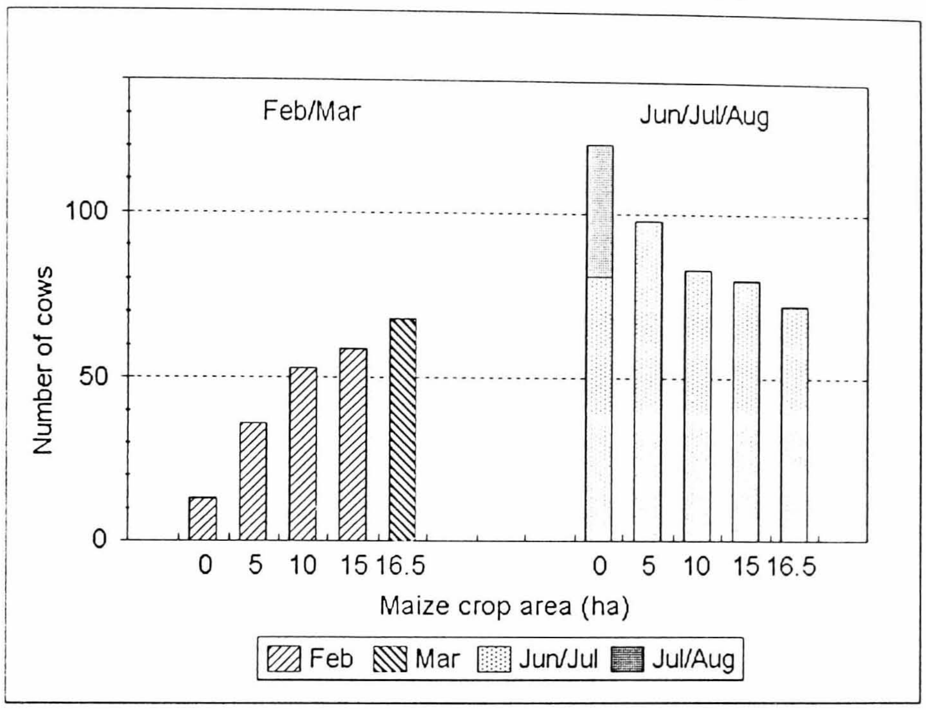
The marginal price of milk quota also increased with larger maize areas, notably for the largest maize area. This indicates that milk quota constraints were more severe on those scenarios with larger maize areas and higher prices for extra milk quota could be paid (Figure 7.2).

**Figure 7.2 - Effects of maize crop on net margin and marginal price of milk quota**



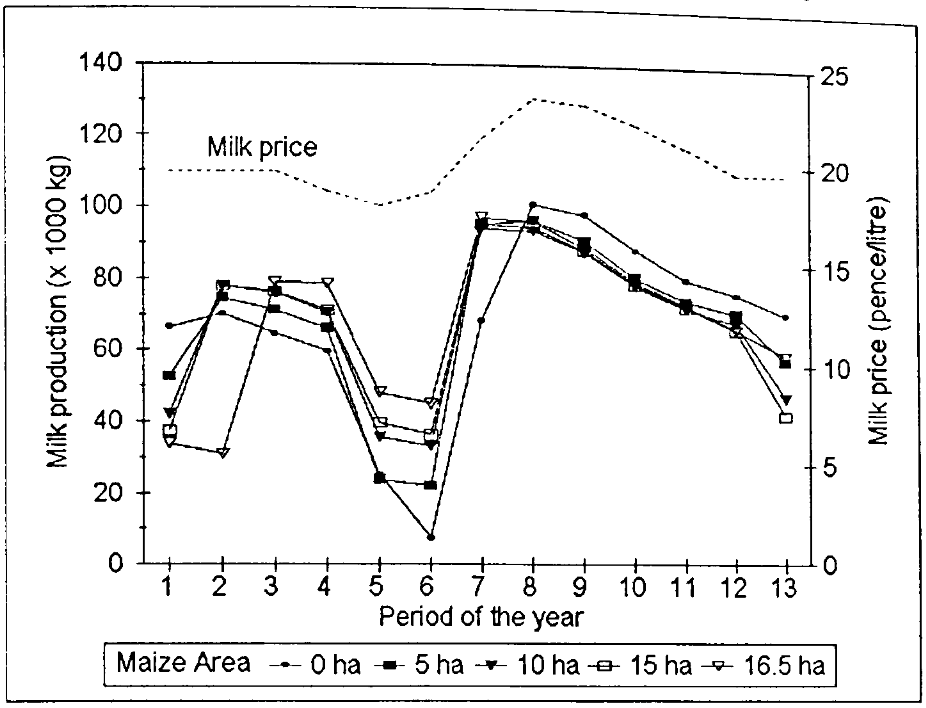
Results show that when the maize crop area was increased, there was a change in the calving pattern from Jun/Jul/Aug to Feb/Mar (from summer milk to spring milk). If these periods are analysed separately, it can be seen that as maize crop area increased more of the cows calved in March rather than February and in Jun/Jul rather than Jul/Aug (Figure 7.3).

**Figure 7.3 - Effects of maize crop: calving pattern**




The total milk production over the year changed when the maize crop area was increased as a consequence of the changes which occurred in the calving pattern. Milk production in spring was increased. During this period milk prices are lower, but the costs of producing milk are also lower. Milk production in Summer, when both milk prices and production costs are higher, was maintained at about the same level (Figure 7.4) by feeding high levels.

Figure 7.4 - Effects of maize crop: total milk production over the year and milk prices




## *Discussion*

When no maize crop was allowed, sixteen hectares of cash crop were grown. When the maize crop area was increased, the cash crop was replaced by the maize crop, while the grass area remained almost the same. Consequently the total forage area increased.

The replacement of concentrates by maize silage showed that the latter helps to reduce the cost of milk production and suggests that maize growers are right when they say that "feeding maize silage should be a concentrate saving exercise" (Mr. Gordon Newman, vice-chairman of Maize Growers' Association, *Farmers Weekly*, 23 September 1994).

The change of calving pattern indicates that it is worth producing more milk in Mar/Apr, even with prices at that time being lower, as this can be justified by the lower costs of production. During their peak milk yield, when energy requirements are higher, cows can graze very good quality grass and have their ration supplemented with maize silage, which is high in energy and cheaper than concentrates. Furthermore, a shift of calving from Jul/Aug to Jun/Jul enables the dairy farmer to achieve higher milk prices and indicates that maize silage is a good complement to Summer grazing.

## 7.3. Effects of milk quota

Milk quota was the major factor limiting production for all the scenarios analysed.

Table 7.2 shows the results of the scenarios when milk quota was varied from low (630 000 litres) to very high (1100 000 litres). The figures for milk quota were based on studies by Nix (1995) with an average annual milk yield of 5500 litres/cow, for stocking rates at low (1.65 cow/ha), average (1.90 cow/ha), high (2.20 cow/ha) and very high (2.50 cow/ha) levels. The scenario with the highest milk quota was calculated based on a extremely high stocking rate (3 cow/ha).

**Table 7.2 - Effects of different milk quotas: summary of results**

| Milk quota (x100 000 l) | Net margin (£) | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| With no maize | | | |
| 6.3 | 73 960 | 8.40 | 6400 |
| 7.3 | 82 723 | 8.30 | 6380 |
| 8.5 | 93 100 | 8.24 | 6351 |
| 9.5 | 101 502 | 7.90 | 6279 |
| 11.0 | 112 931 | 6.51 | 6246 |
| With maize (10 ha) | | | |
| 6.3 | 81 797 | 9.63 | 6077 |
| 8.5 | 101 850 | 8.68 | 6227 |
| 9.5 | 110 053 | 8.45 | 6343 |

| | Average annual consumption (tDM/cow) | | | | | |
|---|---|---|---|---|---|---|
| Milk quota (x 100 000 l) | Grass Silage | Maize Silage | Concentrates | Calving Pattern (Number of cows) | | |
| With no maize | | | | | | |
| 6.3 | 1.71 | 0.00 | 2.26 | | Jun/Jul: 90 | Jul/Aug: 9 |
| 7.3 | 1.53 | 0.00 | 2.36 | Feb: 6 | Jun/Jul: 96 | Jul/Aug: 13 |
| 8.5 | 1.46 | 0.00 | 2.39 | Feb: 13 | Jun/Jul: 81 | Jul/Aug: 40 |
| 9.5 | 1.43 | 0.00 | 2.34 | Feb: 26 | Jun/Jul: 29 | Jul/Aug: 97 |
| 11.0 | 1.42 | 0.00 | 2.37 | Mar: 37 | Jun/Jul: 55 | Jul/Aug: 85 |
| With maize (10 ha) | | | | | | |
| 6.3 | 1.77 | 1.01 | 1.12 | Mar: 26 | Jun/Jul: 78 | |
| 8.5 | 1.62 | 0.77 | 1.57 | Feb: 53 | Jun/Jul: 83 | |
| 9.5 | 1.46 | 0.71 | 1.81 | Feb: 51 | Jun/Jul: 99 | |

Figure 7.5 and Figure 7.6 show the consumption of silage and concentrates when milk quota increases for those scenarios with no maize and with 10 ha of maize crop, respectively.

**Figure 7.5 - Effects of different milk quotas: silage and concentrates consumption and average annual milk yield per cow**

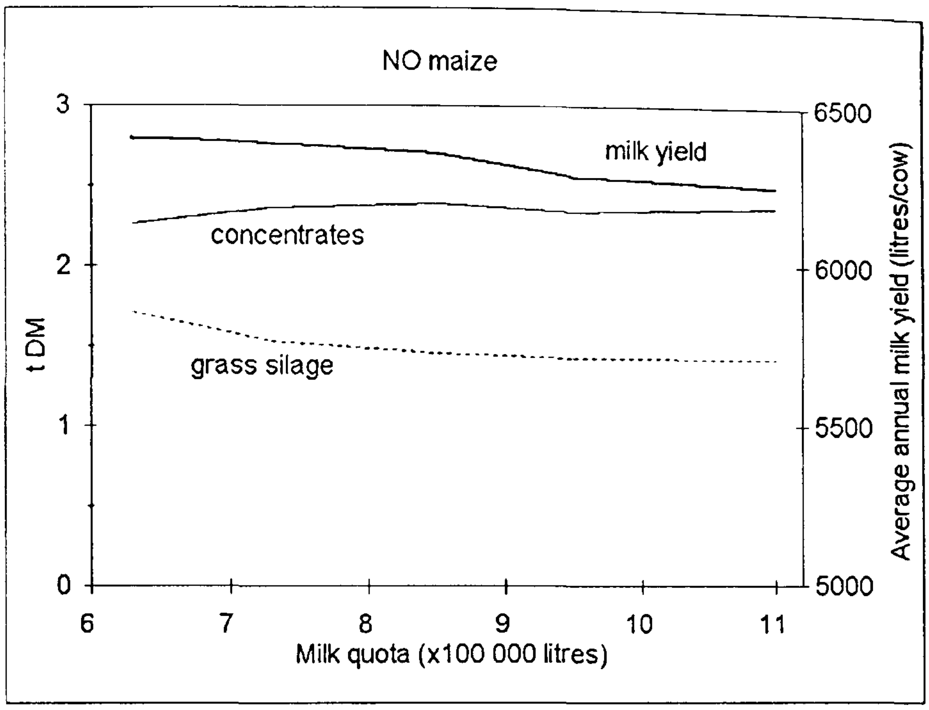
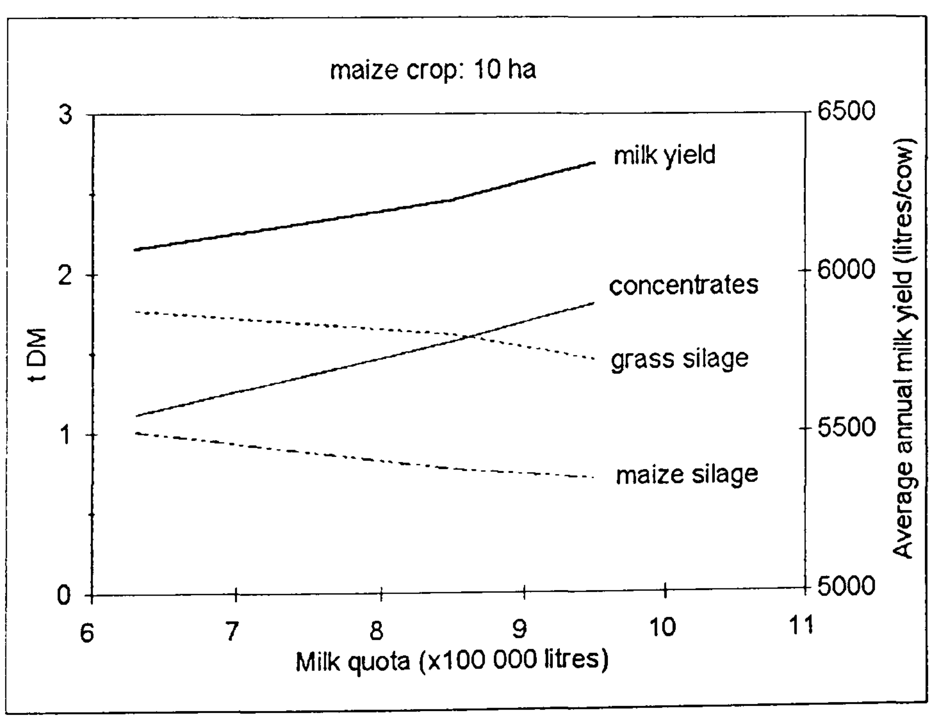


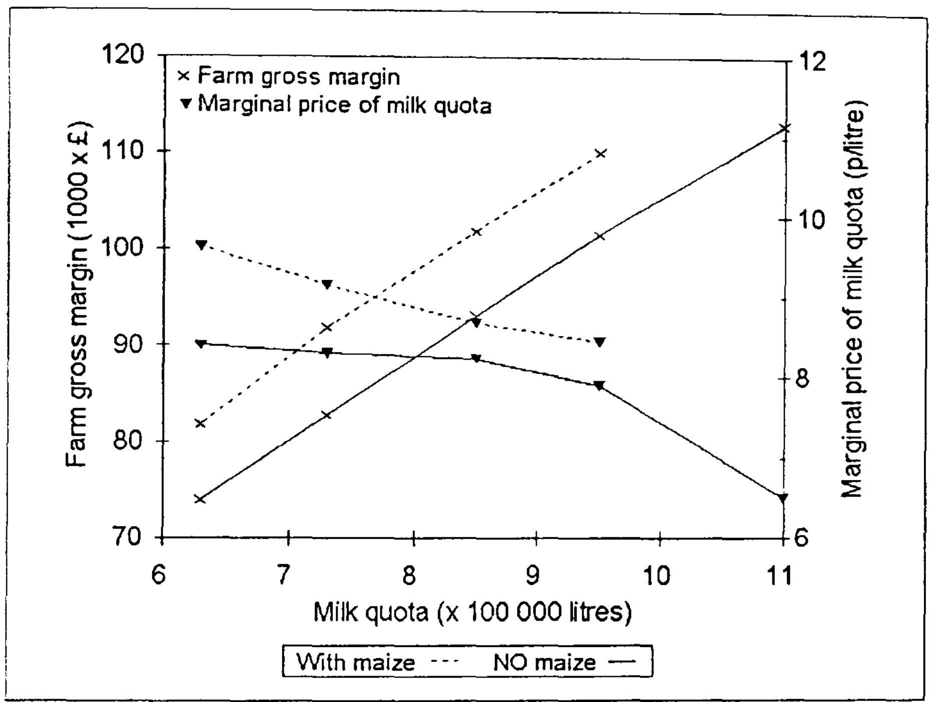

**Figure 7.6 - Effects of different milk quotas: silage and concentrates consumption and average annual milk yield per cow**




When maize silage was available, this replaced concentrates and the consumption of grass silage was slightly reduced. The average annual milk yield per cow decreased when no maize was available and increased when 10 ha of maize were grown. The consumption of concentrates increased with milk quota when maize silage was available yet stayed almost constant when there was no maize.

When milk quota increased, the net margin increased similarly for scenarios with and without maize crop, the former having higher values. Marginal price of milk quota, which was obtained from the shadow price of the milk quota constraint, was lower when milk quotas were higher. When milk quota was extremely high (1100 000 litres), the marginal price was reduced drastically, indicating how critical this constraint is under such circumstances. Results also showed that when maize is available one can afford to pay higher prices for extra milk quota. In these scenarios, the effect of milk quota on the marginal price of milk quota was higher, especially for lower levels of milk quota (Figure 7.7).

**Figure 7.7 - Effects of different milk quotas: net margin and marginal price of milk quota**




The calving pattern changed differently with the increasing of milk quota for those scenarios with no maize compared to those with maize. In both cases the herd size increased with milk quota. When there was no maize available, more cows calved in February and in Jul/Aug, with a reduction of cows calving in Jun/Jul. When maize was available, the number of cows calving in February also increased, while the number of cows calving in Jun/Jul increased (Figure 7.8 and Figure 7.9).

**Figure 7.8 - Effects of different milk quotas: calving pattern (no maize)**

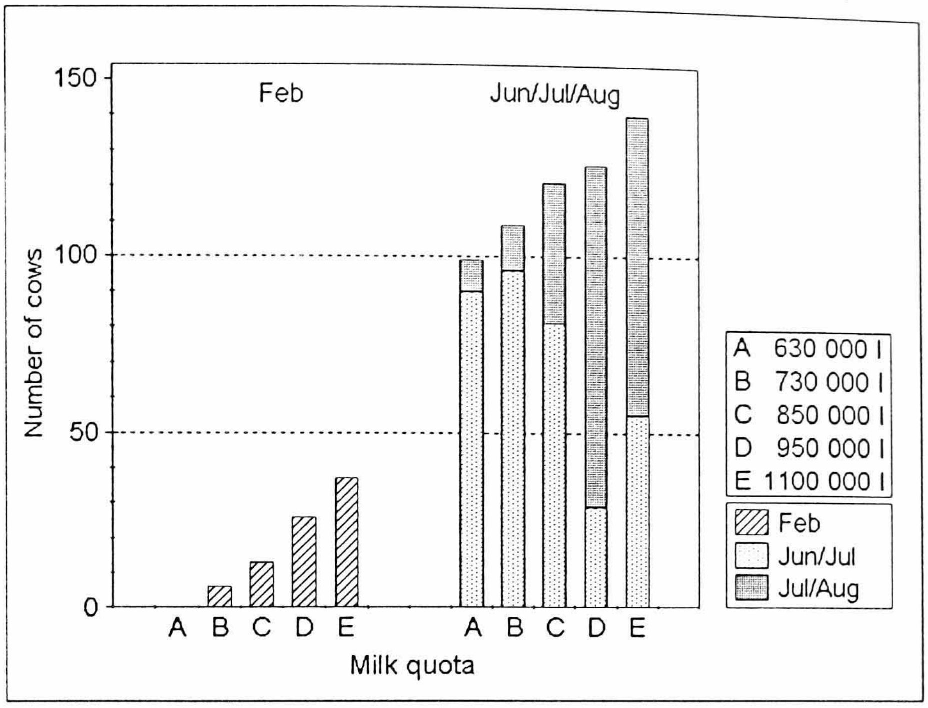
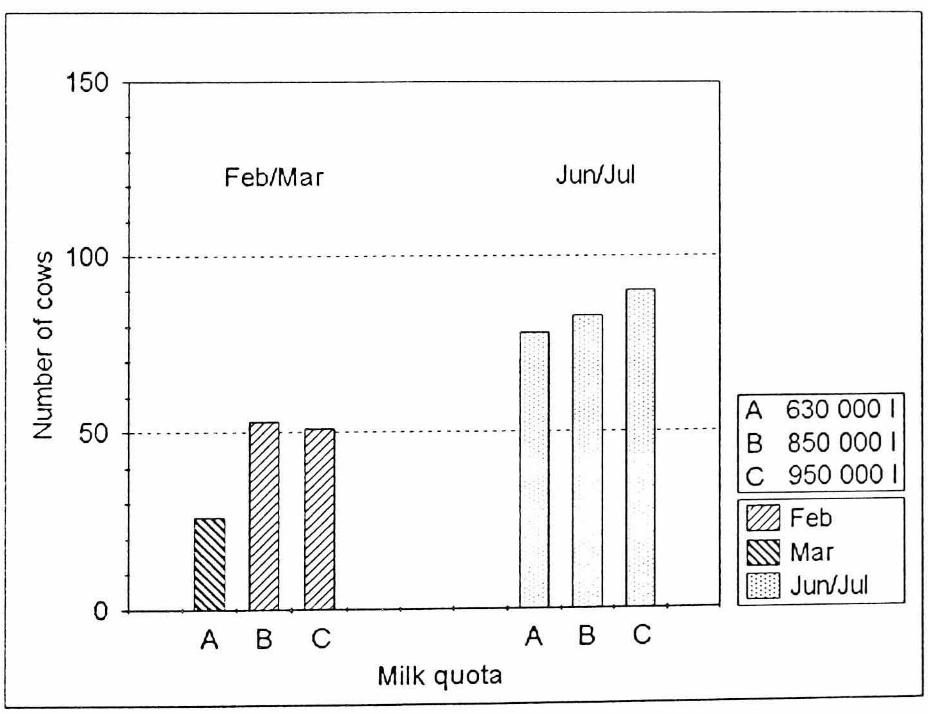



**Figure 7.9 - Effects of different milk quotas: calving pattern (10 ha maize crop)**




The total milk production over the year increased with higher milk quotas. (Figure 7.10 and Figure 7.11). However, because the calving pattern has not changed very much, the milk production pattern has not changed very much either. The peak of Summer milk was slightly delayed as a consequence of the move of the calving pattern from Jun/Jul to Jul/Aug.

Figure 7.10 - Effects of different milk quotas: total milk production over the year (no maize)

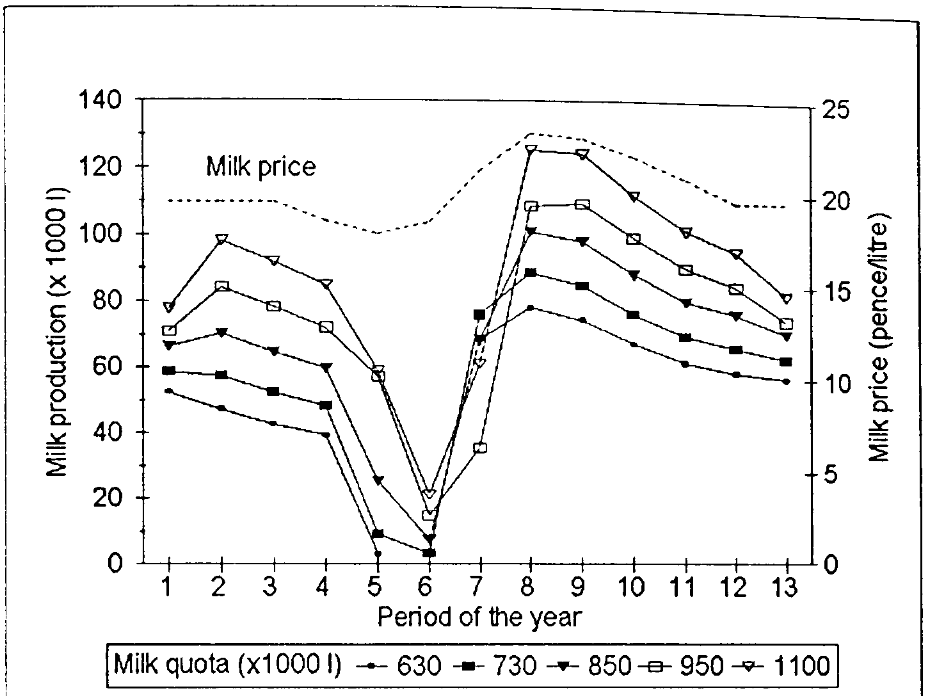
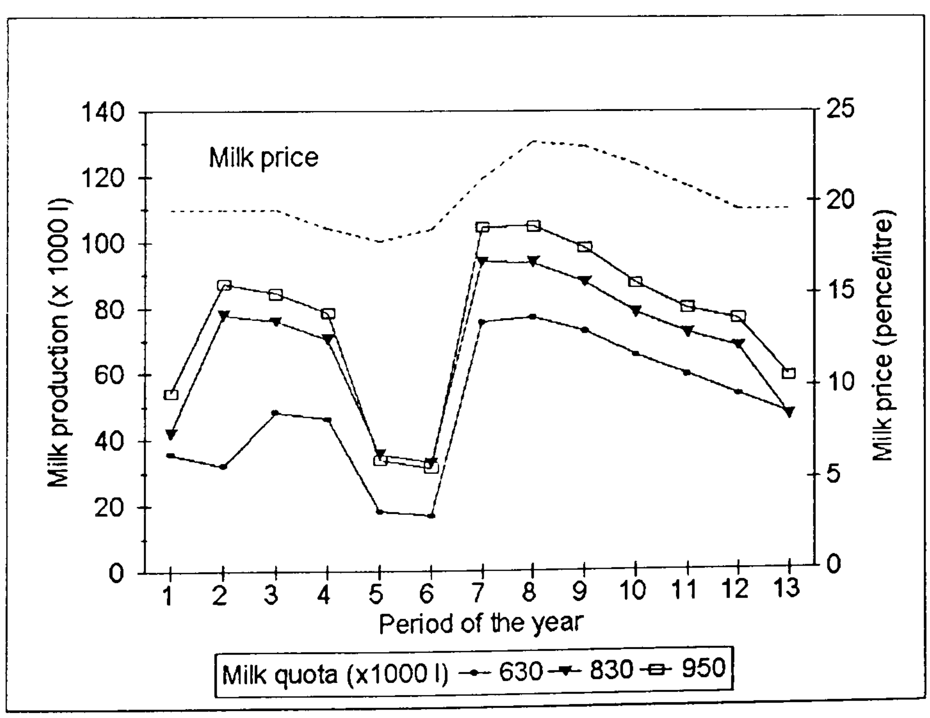



Figure 7.11 - Effects of different milk quotas: total milk production over the year (10 ha maize crop)




## *Discussion*

Since the milk quota was the major factor limiting milk production, herd sizes increased with higher milk quotas. If the predicted results of this model were projected on farms where no maize was available, grass area would increase and cash crop area would decrease with higher milk quotas. Where the maize crop was grown, it almost replaced the cash crop. On those scenarios without maize and with very high quota, all land was allocated to grass. Herd size increased proportionally more than the forage area so consequently the stocking rates were higher with higher milk quotas.

When maize silage was available, the increase of milk quota caused a higher consumption of concentrates and average milk yield per cow. Without maize, consumption of concentrates was about the same for all levels of milk quota, while the consumption of grass silage and the average milk yield decreased. However, this reduction of milk yield was compensated for by a larger number of cows, with the total milk production achieving the quota.

Results show that milk quota had different effects on scenarios with and without maize, concerning energy feeding and milk yield levels. For scenarios without maize, more cows increased the total milk production, but with lower yield, when quota increased. When maize was available, the effect of milk quota was higher and the number of cows and average milk yield increased when quota increased.

When no maize silage was available, the calving pattern changed gradually from Jun/Jul to Jul/Aug and the number of cows calving in February also increased. Cows calving in Jul/Aug consume more concentrates during their peak milk yield, but on the other hand they will be at the end of their lactation, when a lower level of energy is required, during next Spring. This means that they will not need to graze large areas and consequently a larger area of grass can be conserved. Cows calving in Jul/Aug have their peak milk yield during the first half of September, when milk prices are still high. Cows calving in February will produce more milk during the period with lower prices, but on the other hand, they will be able to graze grass of very good quality during their peak milk yield, when energy requirements are higher.

## 7.4. Effects of change of the basic milk price

The basic milk price is calculated according to the milk composition and milk compositional prices. For the standard systems 1 and 2 the basic milk price calculated was 20.3 p/litre. In order to examine the effects of milk prices on the strategic decisions of dairy farms, different scenarios were created for farms under different conditions (low or high milk quota, and with no maize or with 10 ha maize crop) with basic milk prices varying from 18 p/litre to 22 p/litre (± 10% of the price calculated for the standard systems).

Table 7.3 and Table 7.4 summarize the predicted results for farms with low milk quota (630 000 litres) and high milk quota (950 000 litres), respectively.

**Table 7.3 - Effects of changes of the basic milk price: summary of results (low milk quota)**

| Basic milk price (p/litre) | Net margin (£) | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| With no maize | | | |
| 18.0 | 59 539 | 6.18 | 6400 |
| 20.3 | 73 960 | 8.40 | 6400 |
| 22.0 | 84 728 | 10.06 | 6400 |
| With maize (10 ha) | | | |
| 18.0 | 67 739 | 7.40 | 6030 |
| 20.3 | 81 797 | 9.63 | 6030 |
| 22.0 | 92 569 | 11.29 | 6030 |

| Basic milk price (p/litre) | Average annual consumption (tDM/cow) Grass Silage | Maize Silage | Concentrates | Calving Pattern (Number of cows) |
|---|---|---|---|---|
| With no maize | | | | |
| 18.0 | 1.71 | 0.00 | 2.26 | Jun/Jul: 90 Jul/Aug: 9 |
| 20.3 | 1.71 | 0.00 | 2.26 | Jun/Jul: 90 Jul/Aug: 9 |
| 22.0 | 1.71 | 0.00 | 2.26 | Jun/Jul: 90 Jul/Aug: 9 |
| With maize (10 ha) | | | | |
| 18.0 | 1.77 | 1.01 | 1.12 | Mar: 26 Jun/Jul: 78 |
| 20.3 | 1.77 | 1.01 | 1.12 | Mar: 26 Jun/Jul: 78 |
| 22.0 | 1.77 | 1.01 | 1.12 | Mar: 26 Jun/Jul: 78 |

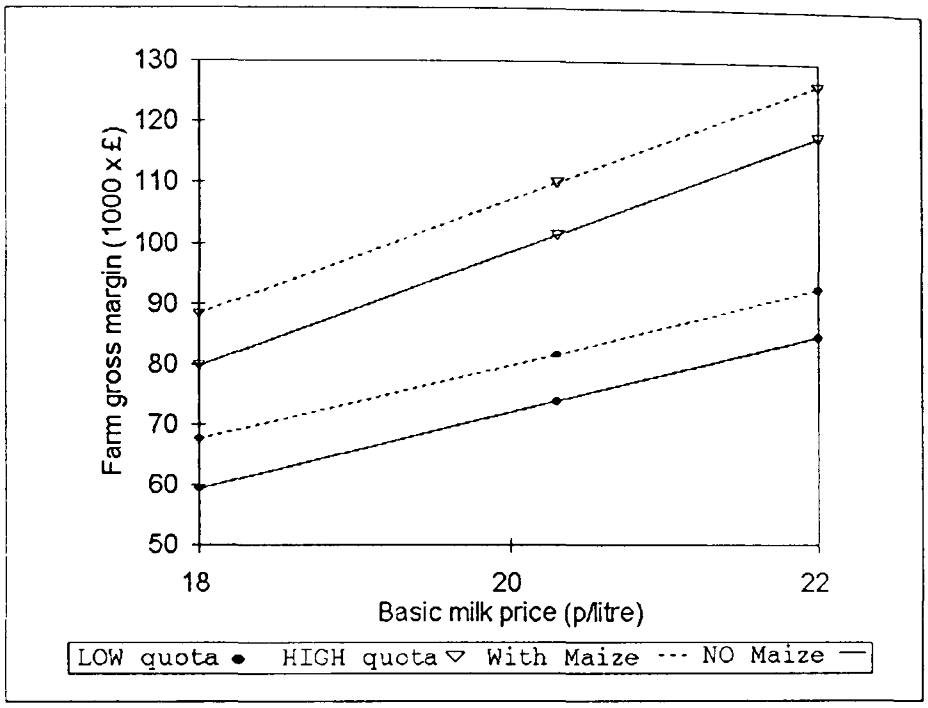
**Table 7.4 - Effects of changes of the basic milk price: summary of results (high milk quota)**

| Basic milk price (p/litre) | Net margin (£) | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| With no maize | | | |
| 18.0 | 79 763 | 5.67 | 6279 |
| 20.3 | 101 502 | 7.90 | 6279 |
| 22.0 | 117 745 | 9.56 | 6279 |
| With maize (10 ha) | | | |
| 18.0 | 88 311 | 6.23 | 6343 |
| 20.3 | 110 053 | 8.45 | 6343 |
| 22.0 | 126 296 | 10.11 | 6343 |

| Basic milk price (p/litre) | Average annual consumption (tDM/cow) Grass Silage | Maize Silage | Concentrates | Calving Pattern (Number of cows) |
|---|---|---|---|---|
| With no maize | | | | |
| 18.0 | 1.43 | 0.00 | 2.34 | Feb: 26 Jun/Jul: 29 Jul/Aug: 97 |
| 20.3 | 1.43 | 0.00 | 2.34 | Feb: 26 Jun/Jul: 29 Jul/Aug: 97 |
| 22.0 | 1.43 | 0.00 | 2.34 | Feb: 26 Jun/Jul: 29 Jul/Aug: 97 |
| With maize (10 ha) | | | | |
| 18.0 | 1.46 | 0.71 | 1.81 | Feb: 51 Jun/Jul: 99 |
| 20.3 | 1.46 | 0.71 | 1.81 | Feb: 51 Jun/Jul: 99 |
| 22.0 | 1.46 | 0.71 | 1.81 | Feb: 51 Jun/Jul: 99 |

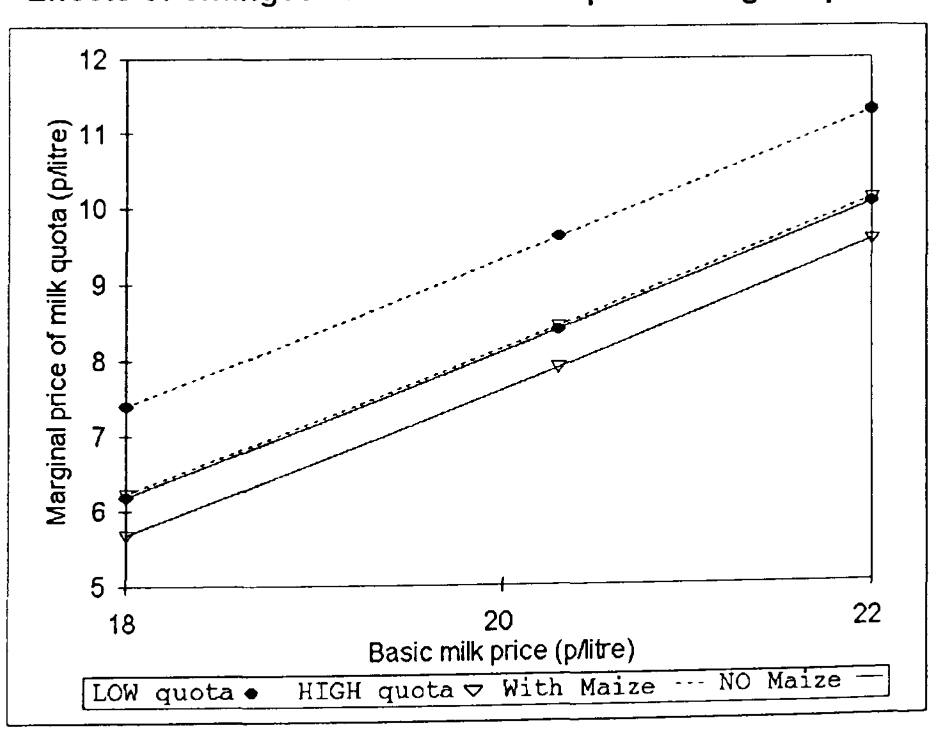
When basic milk prices increased, net margins increased, the effect being markedly higher in those scenarios with higher milk quota (Figure 7.12).

**Figure 7.12 - Effects of changes of the basic milk price: net margin**




The marginal prices of milk quota also increased with higher milk regardless the farm conditions (Figure 7.13).

**Figure 7.13 - Effects of changes of the basic milk price: marginal price of milk quota**



*Discussion*

Results showed that strategic plans did not vary with milk prices. Solutions varied only from the economic point of view. This means that the systems were not sensitive to minor changes in milk prices, although the gross margins and the marginal price of milk quota changed substantially with the changes in milk prices.

## 7.5. Effects of no seasonality of milk prices

The objective of studying these scenarios was to analyse the effects that seasonal changes in milk prices have on dairy farm plans. The seasonal adjustments of milk price aim to stimulate a spread milk production throughout the year. Different scenarios were studied for different conditions such as low or high milk quota and without maize or with 10 ha of maize crop. In order to examine whether different levels of prices would cause any major effect, three levels were used: 18 p/litre, 20 p/litre and 22 p/litre. Table 7.5, with the low quota of 630 000 litres and Table 7.6 with the high quota of 950 000 litres summarize the results for these scenarios.

**Table 7.5 - Effects of no seasonality of milk prices: summary of results (low milk quota)**

| Milk price (p/litre) | Net margin (£) | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| With no maize | | | |
| 18.0 | 57 511 | 6.07 | 6010 |
| 20.0 | 70 100 | 8.01 | 6010 |
| 22.0 | 82 688 | 9.95 | 6010 |
| With maize (10 ha) | | | |
| 18.0 | 65 443 | 7.83 | 5904 |
| 22.0 | 90 621 | 11.71 | 5904 |

| Milk price (p/litre) | Average annual consumption (tDM/cow) Grass Silage | Maize Silage | Concentrates | Calving Pattern (Number of cows) | | | |
|---|---|---|---|---|---|---|---|
| With no maize | | | | | | | |
| 18.0 | 1.68 | 0.00 | 1.77 | Feb: 56 | | | Jul/Aug: 48 |
| 20.0 | 1.68 | 0.00 | 1.77 | Feb: 56 | | | Jul/Aug: 48 |
| 22.0 | 1.68 | 0.00 | 1.77 | Feb: 56 | | | Jul/Aug: 48 |
| With maize (10 ha) | | | | | | | |
| 18.0 | 1.63 | 0.99 | 0.82 | Feb:24 | Mar:44 | Apr:24 | Jul/Aug:14 |
| 22.0 | 1.63 | 0.99 | 0.82 | Feb:24 | Mar:44 | Apr:24 | Jul/Aug:14 |

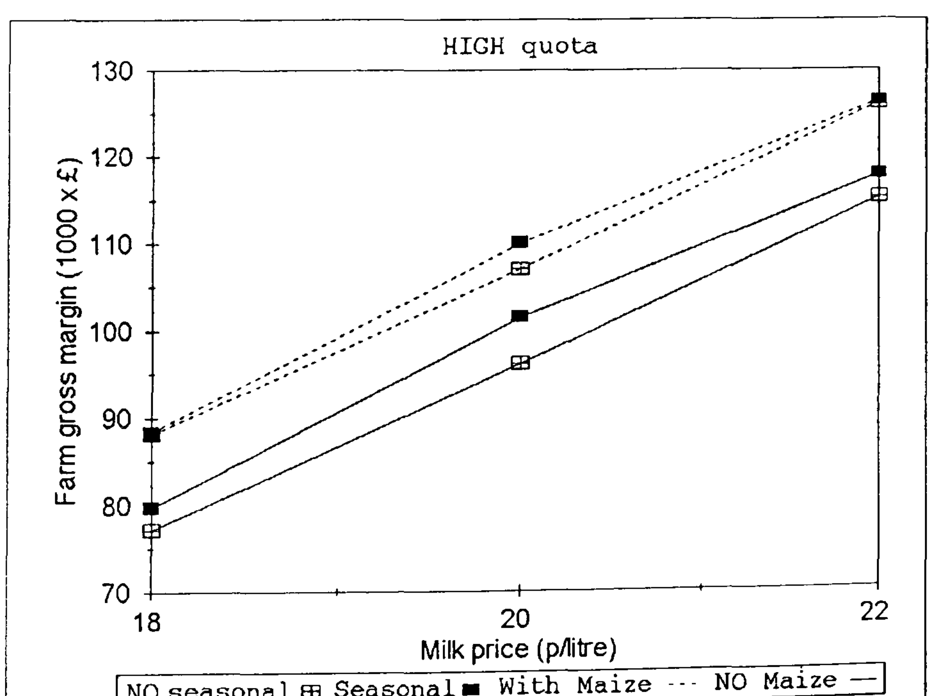
**Table 7.6 - Effects of no seasonality of milk prices: summary of results (high milk quota)**

| Basic milk price (p/litre) | Net margin (£) | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| With no maize | | | |
| 18.0 | 77 174 | 5.89 | 5881 |
| 20.0 | 96 156 | 7.83 | 5881 |
| 22.0 | 115 139 | 9.77 | 5881 |
| With maize (10 ha) | | | |
| 18.0 | 88 106 | 5.50 | 6324 |
| 22.0 | 126 072 | 9.38 | 6324 |

| Basic milk price (p/litre) | Average annual consumption (tDM/cow) Grass Silage | Maize Silage | Concentrates | Calving Pattern (Number of cows) | | |
|---|---|---|---|---|---|---|
| With no maize | | | | | | |
| 18.0 | 1.28 | 0.00 | 2.06 | Feb: 100 | | Jul/Aug: 61 |
| 20.0 | 1.28 | 0.00 | 2.06 | Feb: 100 | | Jul/Aug: 61 |
| 22.0 | 1.28 | 0.00 | 2.06 | Feb: 100 | | Jul/Aug: 61 |
| With maize (10 ha) | | | | | | |
| 18.0 | 1.47 | 0.70 | 1.60 | Feb: 80 | Aug/Sep: 65 | Jul/Aug: 5 |
| 22.0 | 1.47 | 0.70 | 1.60 | Feb: 80 | Aug/Sep: 65 | Jul/Aug: 5 |

Net margins were higher when there were seasonal milk prices than when milk prices were constant throughout the year. Figure 7.14 shows the net margins for farms with high quotas (950 000 litres).

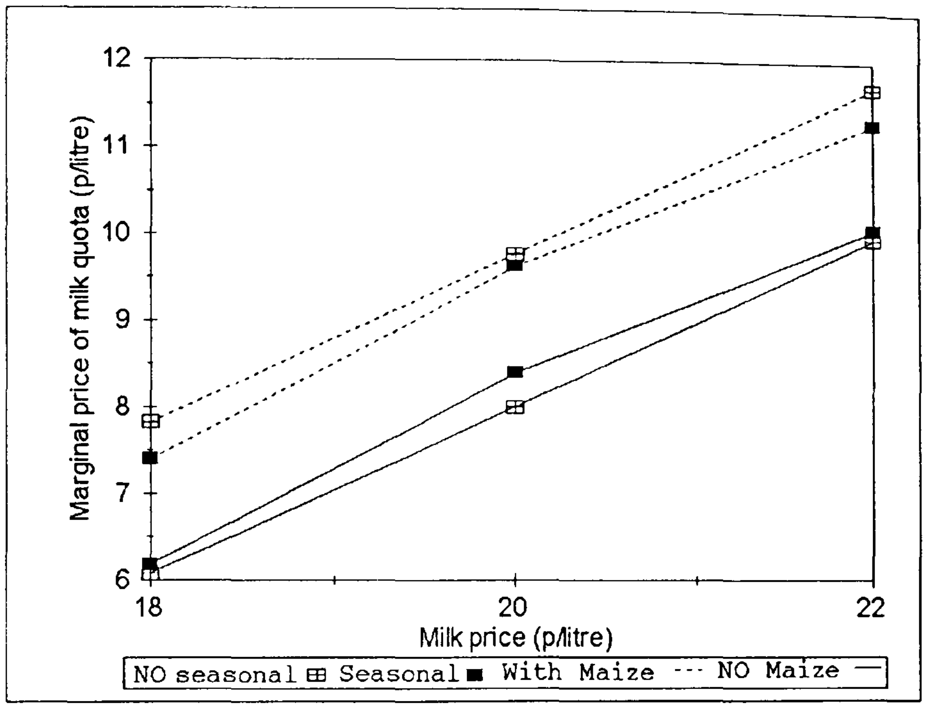
**Figure 7.14 - Effects of no seasonality of milk prices: net margin**




The effect of the seasonality of milk price on the marginal prices of quota were different for scenarios with low and high quotas. With low quotas and maize silage available, marginal prices of quota were higher when milk prices were constant throughout the year.

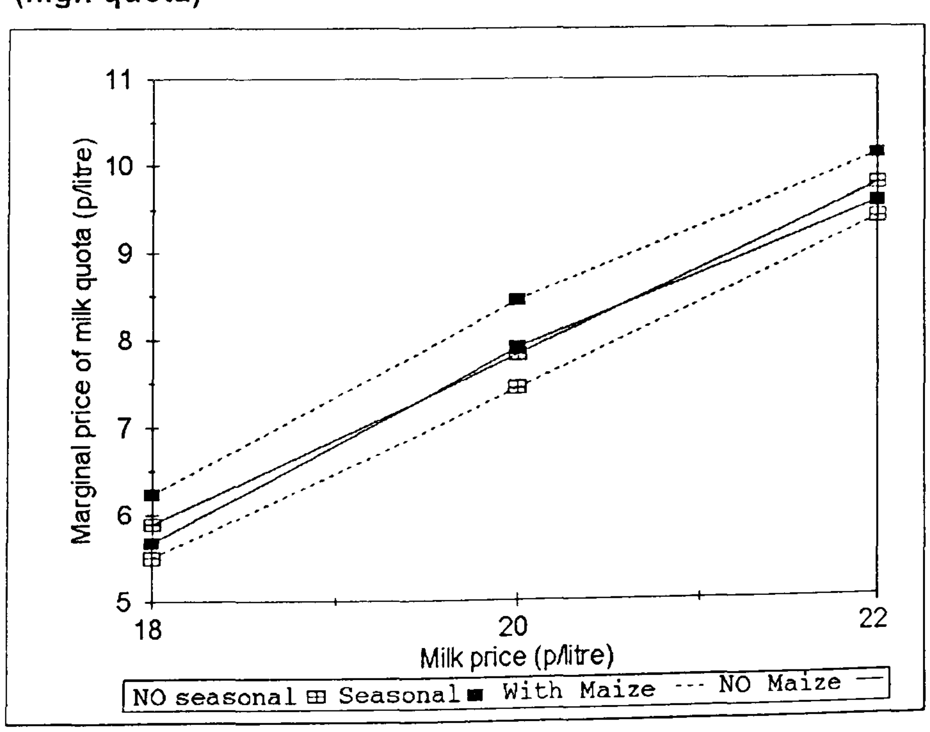
Without maize, marginal prices of quota were lower when milk prices were constant (Figure 7.15).

**Figure 7.15 - Effects of no seasonality of milk prices: marginal price of milk quota (low quota)**




On those scenarios with high quotas, marginal prices of quota were slightly lower when milk prices were constant throughout the year (Figure 7.16).

**Figure 7.16 - Effects of no seasonality of milk prices: marginal price of milk quota (high quota)**



The seasonality of milk prices also affected the feeding levels, especially consumption of concentrates, and milk yield. When milk prices were constant, lower levels of concentrates were consumed; grass silage and maize silage consumption being about the same (Figure 7.17 and Figure 7.18)

**Figure 7.17 - Effects of no seasonality of milk prices: silage and concentrates consumption (low quota)**

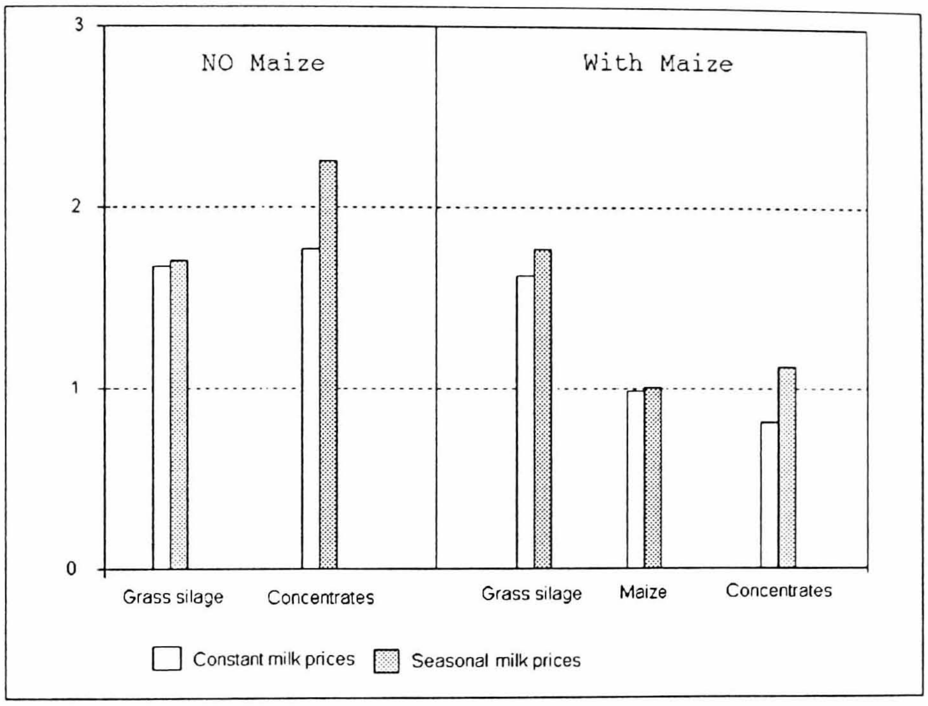
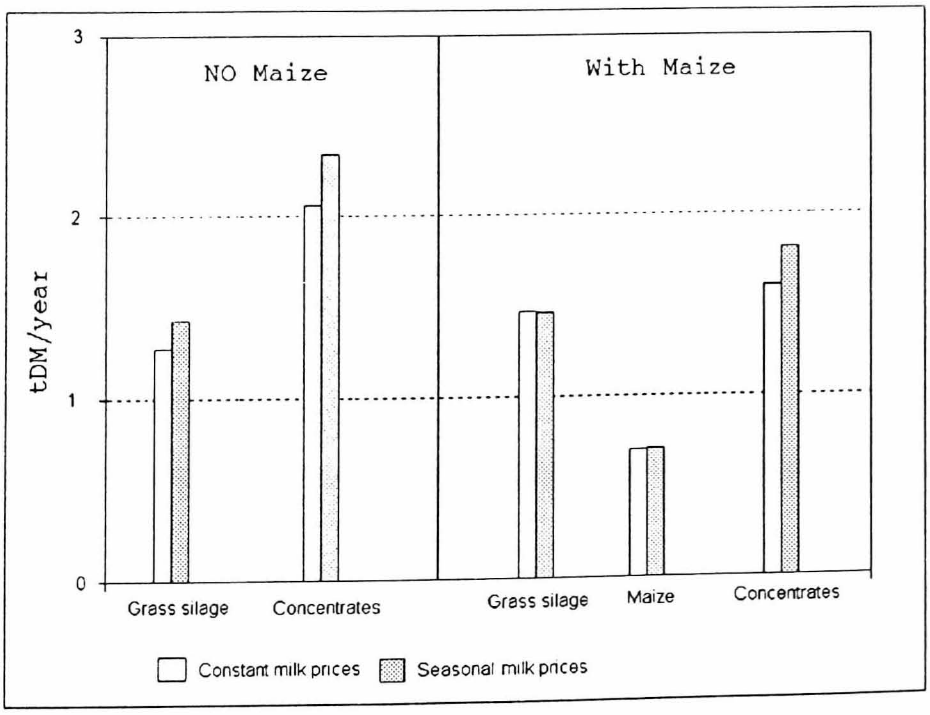
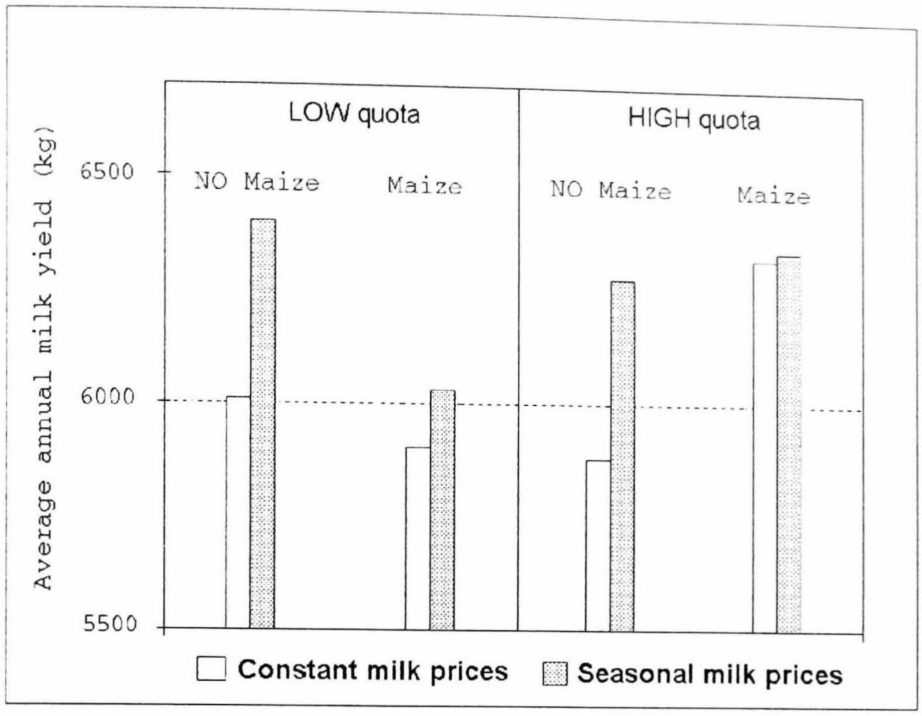



**Figure 7.18 - Effects of no seasonality of milk prices: silage and concentrates consumption (high quota)**




Average milk yields were also higher when milk prices varied throughout the year, especially when maize was not available (Figure 7.19).

**Figure 7.19 - Effects of no seasonality of milk prices: average annual milk yields**



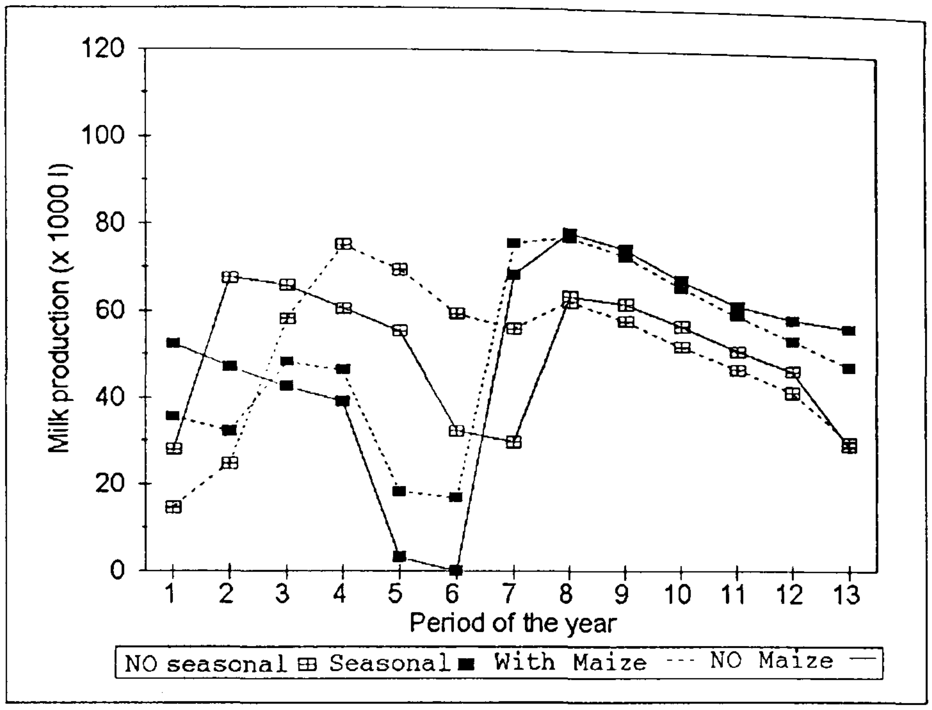

The calving patterns were particularly affected by the seasonality of milk prices. The effects were different for scenarios with or without maize and with low or high quotas.

For scenarios with low quotas and no maize, when prices varied seasonally the calving pattern was concentrated in Jun/Jul and Jul/Aug (90 and 9 cows, respectively). When prices were constant, cows calved in February (56 cows) and Jul/Aug (48 cows). When maize was available and the milk prices constant, the number of cows calving during Summer was drastically reduced (from 78 in Jun/Jul to 14 in Jul/Aug) and the number of Spring calves increased (26 in March to 24 in February, 44 in March and 24 in April).

For scenarios with high quota and no maize, constant milk prices caused a higher number of cows calving in February (from 26 to 100) and a reduction in Summer calves (from 29 in Jun/Jul and 97 in Jul/Aug to 61 in Jul/Aug). When maize was available, constant milk prices caused a higher number of cows calving in February (from 51 to 80) and a shift from Summer calving to Autumn and Winter calving (from 99 cows in Jun/Jul to 65 in Aug/Sep and 5 in December).

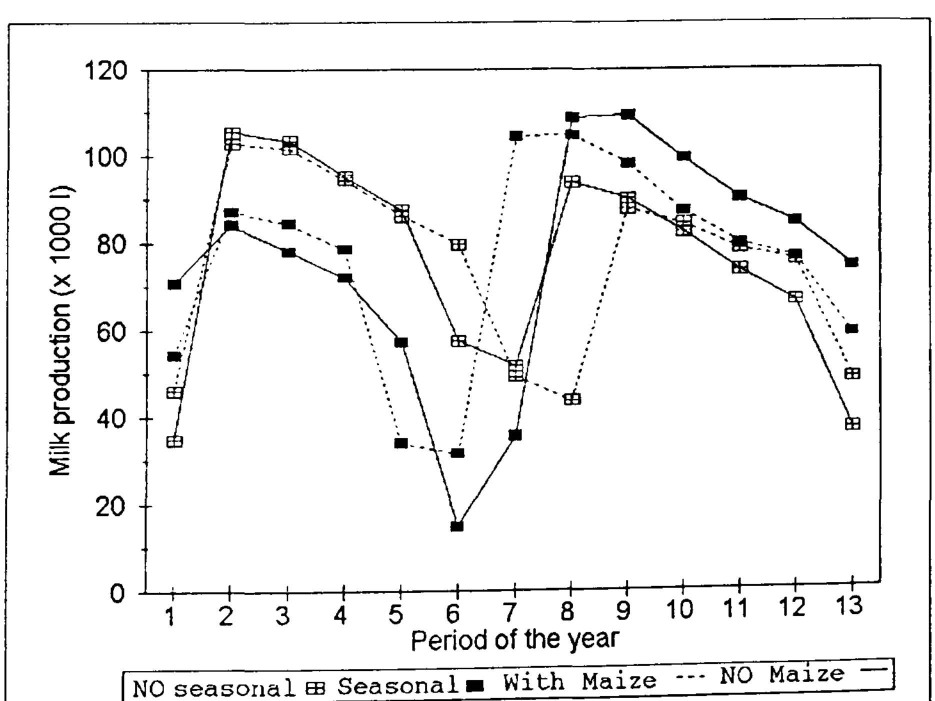
As a consequence of the changes in the calving pattern due to the seasonality of milk prices, total milk production over the year was also affected. For scenarios with low quota, higher summer milk prices stimulated higher milk production during that period and lower production when milk prices were lower (Spring). When milk prices were constant throughout the year, the milk production was more equally distributed throughout the year, with a slight peak during Spring, when production costs are lower due to good quality grass for grazing (Figure 7.20).

**Figure 7.20 - Effects of no seasonality of milk prices: total milk production over the year (low quota)**




When milk quota was high, the seasonality of prices also affected the milk production throughout the year. With seasonal milk prices, the two peaks of milk production were in Spring and Summer/Autumn, the latter being considerably higher. With constant milk prices, production was spread more evenly throughout the year with peaks in Spring and Summer (Figure 7.21).

**Figure 7.21 - Effects of no seasonality of milk prices: total milk production over the year (high quota)**



## 7.6. Effects of changes of the price of concentrates

Concentrates are the most expensive component of the dairy ration so this group of scenarios investigated how the price of concentrates affects the strategic plans of dairy farms. The price of concentrates for the standard systems 1 and 2 is £ 155 /tDM. Different scenarios were studied having lower or higher milk quota and with or without maize crop and price of concentrates varying from £ 140 /tDM or £ 170 /tDM (± 10% of the price for the standard systems).

Table 7.7 and Table 7.8 show the results with two different prices for milk (18 p/litre and 22 p/litre) for scenarios with lower (630 000 litres) and higher (950 000 litres) milk quotas, respectively.

**Table 7.7 - Effects of changes of the price of concentrates: summary of results (low milk quota)**

| Concentrate price (£/tDM) | Net margin (£) | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| | Basic milk price: 18 p/litre | | |
| With no maize | | | |
| 140 | 62 967 | 6.98 | 6405 |
| 155 | 59 539 | 6.18 | 6400 |
| 170 | 56 368 | 5.51 | 6266 |
| With maize (10 ha) | | | |
| 140 | 69 211 | 8.09 | 6077 |
| 155 | 67 379 | 7.40 | 6030 |
| 170 | 65 548 | 6.67 | 5944 |
| | Basic milk price: 22 p/litre | | |
| With no maize | | | |
| 140 | 88 156 | 10.87 | 6405 |
| 155 | 84 728 | 10.06 | 6400 |
| 170 | 81 556 | 9.39 | 6266 |
| With maize (10 ha) | | | |
| 140 | 94 401 | 11.97 | 6077 |
| 155 | 92 569 | 11.29 | 6030 |
| 170 | 90 739 | 10.55 | 5944 |

| Concentrate price (£/tDM) | Average annual consumption (tDM/cow): Grass Silage | Maize Silage | Concentrates | Calving Pattern (Number of cows) | | |
|---|---|---|---|---|---|---|
| | | Basic milk price: 18 p/litre | | | | |
| With no maize | | | | | | |
| 140 | 1.70 | 0.00 | 2.29 | | Jun/Jul: 98 | Jul/Aug: 1 |
| 155 | 1.71 | 0.00 | 2.26 | | Jun/Jul: 90 | Jul/Aug: 9 |
| 170 | 1.88 | 0.00 | 2.02 | Feb: 6 | Jun/Jul: 29 | Jul/Aug: 65 |
| With maize (10 ha) | | | | | | |
| 140 | 1.71 | 1.02 | 1.15 | Mar: 23 | Jun/Jul: 81 | |
| 155 | 1.77 | 1.01 | 1.12 | Mar: 26 | Jun/Jul: 78 | |
| 170 | 1.77 | 1.00 | 1.04 | Mar: 33 | Jun/Jul: 73 | |
| | | Basic milk price: 22 p/litre | | | | |
| With no maize | | | | | | |
| 140 | 1.70 | 0.00 | 2.29 | | Jun/Jul: 98 | Jul/Aug: 1 |
| 155 | 1.71 | 0.00 | 2.26 | | Jun/Jul: 90 | Jul/Aug: 9 |
| 170 | 1.88 | 0.00 | 2.02 | Feb: 6 | Jun/Jul: 29 | Jul/Aug: 65 |
| With maize (10 ha) | | | | | | |
| 140 | 1.71 | 1.02 | 1.15 | Mar: 23 | Jun/Jul: 81 | |
| 155 | 1.77 | 1.01 | 1.12 | Mar: 26 | Jun/Jul: 78 | |
| 170 | 1.77 | 1.00 | 1.04 | Mar: 33 | Jun/Jul: 73 | |

**Table 7.8 - Effects of changes of the price of concentrates: summary of results (high milk quota)**

| Concentrate price (£/tDM) | Net margin (£) | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| | Basic milk price: 18 p/litre | | |
| With no maize | | | |
| 140 | 85 333 | 6.11 | 6304 |
| 155 | 79 763 | 5.67 | 6279 |
| 170 | 74 422 | 5.23 | 6216 |
| With maize (10 ha) | | | |
| 140 | 93 145 | 7.03 | 6388 |
| 155 | 88 311 | 6.23 | 6343 |
| 170 | 84 214 | 5.40 | 6330 |
| | Basic milk price: 22 p/litre | | |
| With no maize | | | |
| 140 | 123 315 | 9.99 | 6304 |
| 155 | 117 745 | 9.56 | 6279 |
| 170 | 112 405 | 9.12 | 6216 |
| With maize (10 ha) | | | |
| 140 | 131 120 | 10.91 | 6388 |
| 155 | 126 296 | 10.11 | 6343 |
| 170 | 122 199 | 9.28 | 6330 |

| Concentrate price (£/tDM) | Average annual consumption (tDM/cow) Grass Silage | Maize Silage | Concentrates | Calving Pattern (Number of cows) | | |
|---|---|---|---|---|---|---|
| | | Basic milk price: 18 p/litre | | | | |
| With no maize | | | | | | |
| 140 | 1.43 | 0.00 | 2.39 | Feb: 25 | Jun/Jul: 55 | Jul/Aug: 71 |
| 155 | 1.43 | 0.00 | 2.34 | Feb: 26 | Jun/Jul: 29 | Jul/Aug: 97 |
| 170 | 1.42 | 0.00 | 2.25 | Feb: 40 | | Jul/Aug: 112 |
| With maize (10 ha) | | | | | | |
| 140 | 1.49 | 0.71 | 1.83 | Feb: 45 | Jun/Jul: 103 | |
| 155 | 1.46 | 0.71 | 1.81 | Feb: 51 | Jun/Jul: 99 | |
| 170 | 1.46 | 0.70 | 1.81 | Feb: 52 | Jun/Jul: 98 | |
| | | Basic milk price: 22 p/litre | | | | |
| With no maize | | | | | | |
| 140 | 1.43 | 0.00 | 2.39 | Feb: 25 | Jun/Jul: 55 | Jul/Aug: 71 |
| 155 | 1.43 | 0.00 | 2.34 | Feb: 26 | Jun/Jul: 29 | Jul/Aug: 97 |
| 170 | 1.42 | 0.00 | 2.25 | Feb: 40 | | Jul/Aug: 112 |
| With maize (10 ha) | | | | | | |
| 140 | 1.49 | 0.71 | 1.83 | Feb: 45 | Jun/Jul: 103 | |
| 155 | 1.46 | 0.71 | 1.81 | Feb: 51 | Jun/Jul: 99 | |
| 170 | 1.46 | 0.70 | 1.81 | Feb: 52 | Jun/Jul: 98 | |

Gross margins were reduced with the increase of price of concentrates, the effects being slightly higher for those scenarios with higher quotas. (Figure 7.22 and Figure 7.23).

Figure 7.22 - Effects of changes of the price of concentrates: net margin (milk price: 18 p/litre)

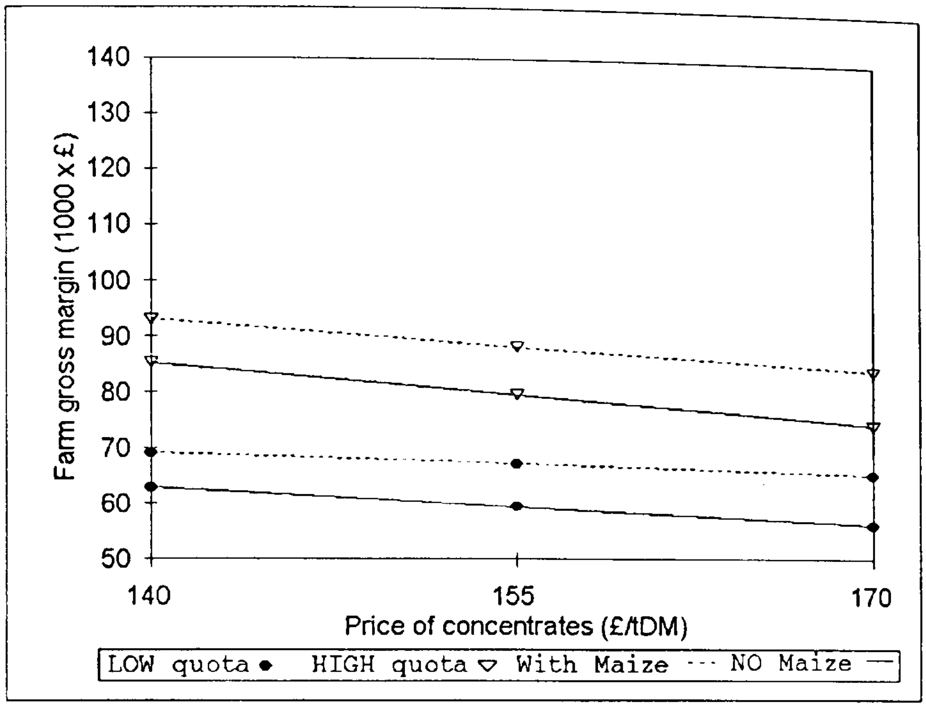
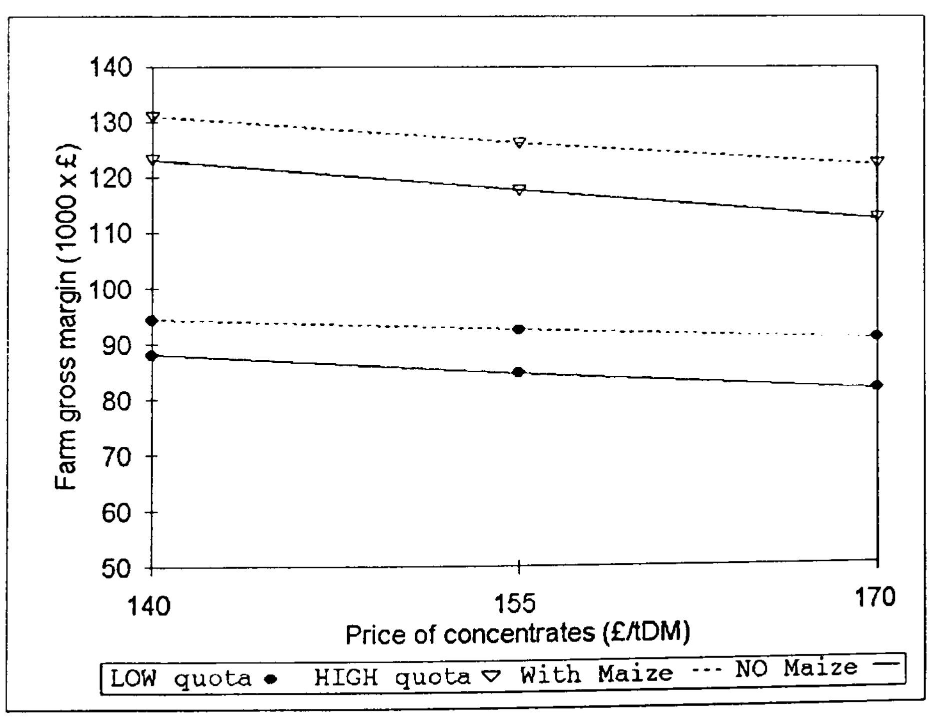
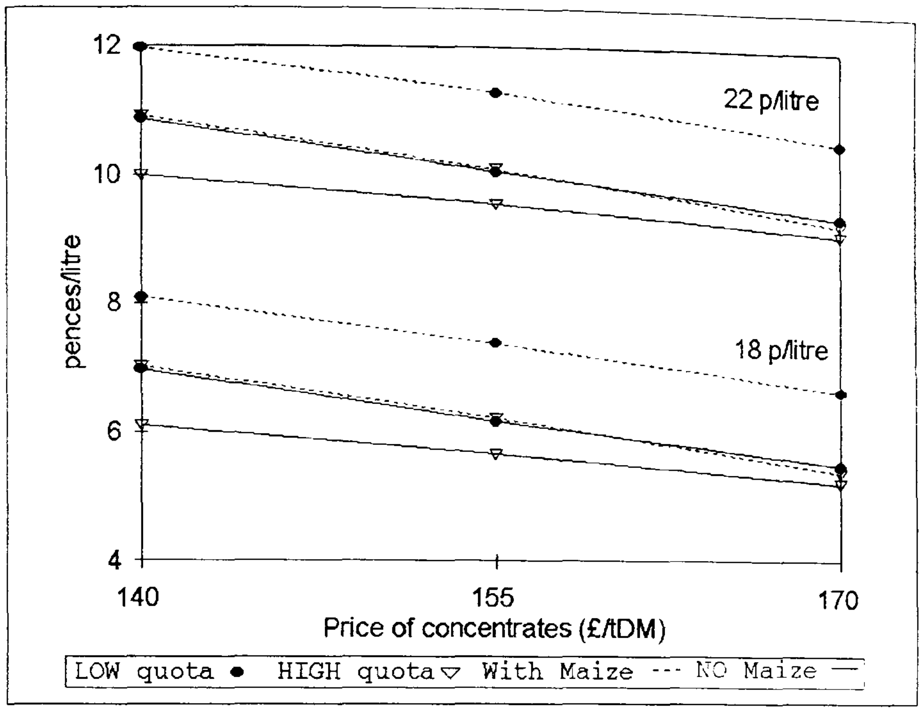



Figure 7.23 - Effects of changes of the price of concentrates: net margin (milk price: 22 p/litre)



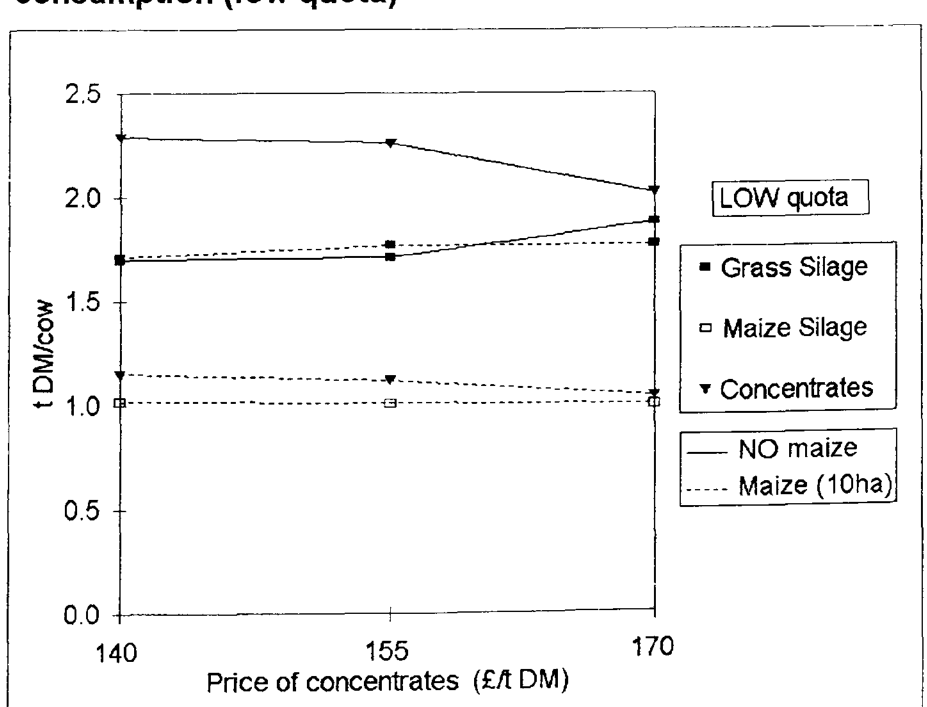

Marginal prices of milk quota also decreased when prices of concentrates increased, the effect being slightly lower for scenarios with higher quotas and no maize (Figure 7.24).

**Figure 7.24 - Effects of changes of the price of concentrates: marginal price of milk quota**



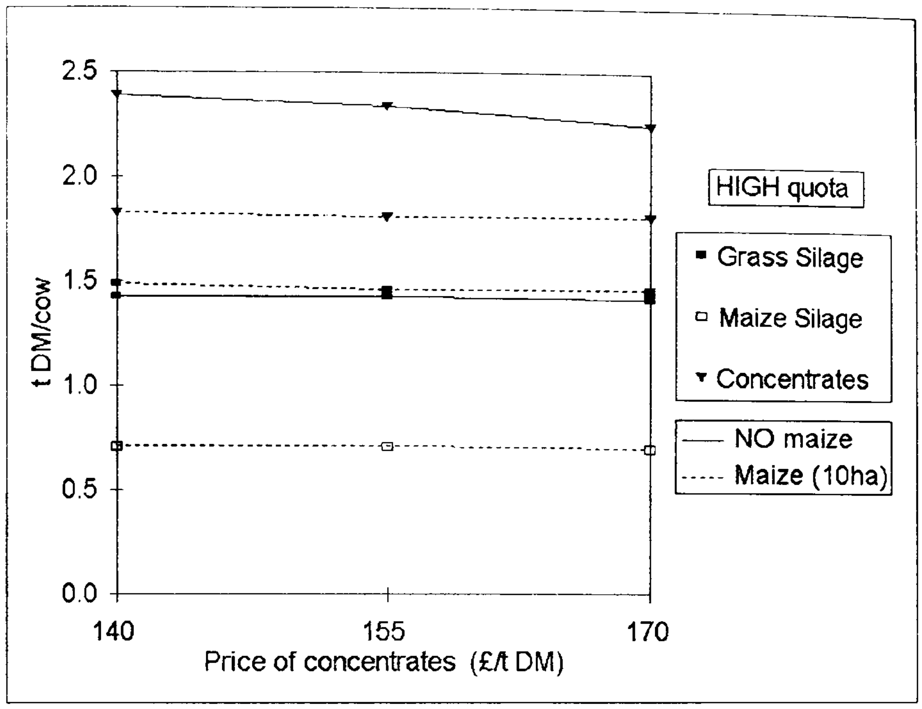

For scenarios with lower milk quotas and no maize available, when the price of the concentrates increased, then grass silage replaced concentrates. The level of consumption of both grass silage and concentrates did not change when the price of concentrates decreased. When prices of concentrates were high, there was a slight reduction in their consumption (Figure 7.25).

**Figure 7.25 - Effects of changes of the price of concentrates: silage and concentrates consumption (low quota)**



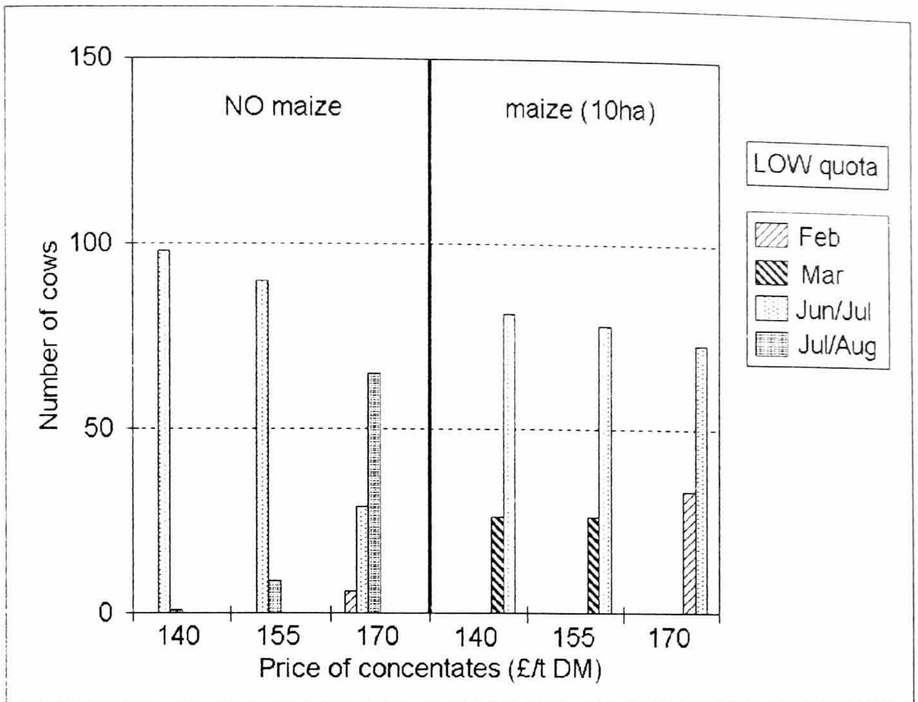
For scenarios with high milk quota, the consumption of grass silage (and maize silage, when available) did not change when the price of concentrates increased. The consumption of concentrates was slightly reduced when their prices increased.

**Figure 7.26 - Effects of changes of the price of concentrates: silage and concentrates consumption (high quota)**



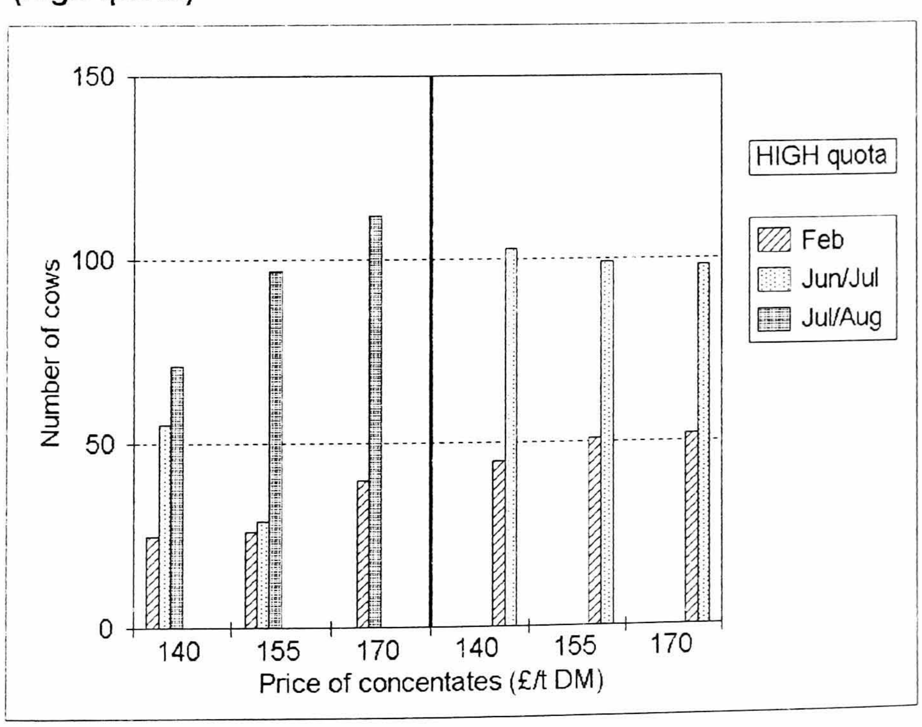

The calving patterns changed differently for those scenarios with lower and higher milk quotas. When milk quota was low and there was no maize available, more cows calved in Jun/Jul (and less in Jul/Aug) when the price of concentrates decreased from £ 155 / tDM to £ 140 / tDM. When the price of concentrates increased to £ 170 / tDM, the calving pattern shifted from mainly Jun/Jul to mainly Jul/Aug with some cows calving in February. When maize silage was available, there was a reduction in the number of cows calving in Jun/Jul and more cows calving in February and March when price of concentrates increased (Figure 7.27).

**Figure 7.27 - Effects of changes of the price of concentrates: calving pattern (low quota)**




For scenarios with higher milk quota and with no maize, when the price of concentrates increased, more cows calved in February and Jul/Aug and less cows calved in Jun/Jul. When maize was available, similar changes occurred with more cows calving in February and less in Jun/Jul, but these changes were less severe (Figure 7.28).

**Figure 7.28 - Effects of changes of the price of concentrates: calving pattern (high quota)**



*Discussion*

Gross margins and marginal prices of milk quota decreased with higher prices of concentrates, since the cost of concentrates has a substantial impact on the total cost of milk production.

Concentrates consumption was affected by their prices only when milk quotas were more critical and maize was not available.

The main effect of the changes of the prices of concentrates was on the calving pattern. With higher prices of concentrates, more cows calved later (Jul/Aug rather than Jun/Jul). Calving at this period, it was still possible to get better milk prices and, because the cows will be at the end of lactation during the following Spring (with lower energy requirements), larger areas of grass can be harvested and better quality grass silage made for winter feeding.

## 7.7. Effects of grazing efficiency

Grazing efficiency may vary from one farm to another for several reasons: different types of soil may cause different levels of damage to grass after cows graze, different management and use of paddocks, restraining cows to smaller areas for different periods.

A grazing efficiency of 60% was assumed for the standard systems 1 and 2 (Parsons, 1993). So, there is a loss of 40% of the DM yield of the grass. In order to analyse the effects of grazing efficiency on dairy farm plans, the optimal plans were compared for scenarios under different conditions with lower or higher milk quota (630 000 litres or 950 000 litres), with or without maize available and different grazing efficiencies (65% and 70%).

Table 7.9 and Table 7.10 show the results with two different prices of milk: 18 p/litre and 22 p/litre.

**Table 7.9 - Effects of grazing efficiency: summary of results (low milk quota)**

| Grazing efficiency (%) | Net margin (£) | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| | Basic milk price: 18 p/litre | | |
| With no maize | | | |
| 60 | 59 539 | 6.18 | 6400 |
| 65 | 60 895 | 6.38 | 6337 |
| 70 | 61 972 | 6.55 | 6258 |
| With maize (10 ha) | | | |
| 60 | 67 379 | 7.40 | 6030 |
| 65 | 68 463 | 7.45 | 6037 |
| 70 | 69 569 | 7.50 | 6102 |
| | Basic milk price: 22 p/litre | | |
| With no maize | | | |
| 60 | 84 728 | 10.06 | 6400 |
| 65 | 86 084 | 10.26 | 6337 |
| 70 | 87 161 | 10.43 | 6258 |
| With maize (10 ha) | | | |
| 60 | 92 569 | 11.29 | 6030 |
| 65 | 93 651 | 11.33 | 6037 |
| 70 | 94 759 | 11.38 | 6102 |

| Grazing efficiency (%) | Average annual consumption (tDM/cow): Grass Silage | Maize Silage | Concentrates | Calving Pattern (Number of cows) | | |
|---|---|---|---|---|---|---|
| | | Basic milk price: 18 p/litre | | | | |
| With no maize | | | | | | |
| 60 | 1.71 | 0.00 | 2.26 | | Jun/Jul: 90 | Jul/Aug: 9 |
| 65 | 1.65 | 0.00 | 2.24 | | Jun/Jul: 90 | Jul/Aug: 9 |
| 70 | 1.61 | 0.00 | 2.20 | Feb: 1 | Jun/Jul: 91 | Jul/Aug: 9 |
| With maize (10 ha) | | | | | | |
| 60 | 1.77 | 1.01 | 1.12 | | Mar: 26 | Jun/Jul: 78 |
| 65 | 1.66 | 1.01 | 1.15 | Feb: 16 | Mar: 3 | Jun/Jul: 86 |
| 70 | 1.63 | 1.02 | 1.19 | Feb: 13 | | Jun/Jul: 90 |
| | | Basic milk price: 22 p/litre | | | | |
| With no maize | | | | | | |
| 60 | 1.71 | 0.00 | 2.26 | | Jun/Jul: 90 | Jul/Aug: 9 |
| 65 | 1.65 | 0.00 | 2.24 | | Jun/Jul: 90 | Jul/Aug: 9 |
| 70 | 1.61 | 0.00 | 2.20 | Feb: 1 | Jun/Jul: 91 | Jul/Aug: 9 |
| With maize (10 ha) | | | | | | |
| 60 | 1.77 | 1.01 | 1.12 | | Mar: 26 | Jun/Jul: 78 |
| 65 | 1.66 | 1.01 | 1.15 | Feb: 16 | Mar: 3 | Jun/Jul: 86 |
| 70 | 1.63 | 1.02 | 1.19 | Feb: 13 | | Jun/Jul: 90 |

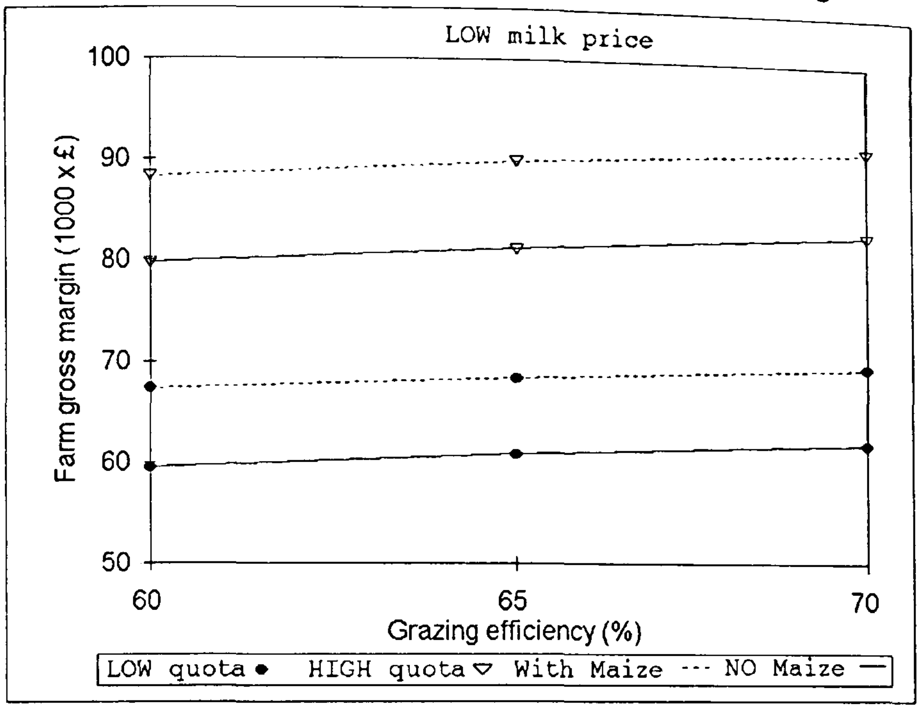
**Table 7.10 - Effects of grazing efficiency: summary of results (high milk quota)**

| Grazing efficiency (%) | Net margin (£) | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| | Basic milk price: 18 p/litre | | |
| With no maize | | | |
| 60 | 79 763 | 5.67 | 6279 |
| 65 | 81 286 | 5.72 | 6218 |
| 70 | 82 735 | 5.59 | 6106 |
| With maize (10 ha) | | | |
| 60 | 88 311 | 6.23 | 6343 |
| 65 | 89 928 | 6.35 | 6359 |
| 70 | 91 334 | 6.52 | 6373 |
| | Basic milk price: 22 p/litre | | |
| With no maize | | | |
| 60 | 117 745 | 9.56 | 6279 |
| 65 | 119 270 | 9.60 | 6218 |
| 70 | 120 717 | 9.48 | 6106 |
| With maize (10 ha) | | | |
| 60 | 126 296 | 10.11 | 6343 |
| 65 | 127 913 | 10.23 | 6359 |
| 70 | 129 319 | 10.40 | 6373 |

| Grazing efficiency (%) | Average annual consumption (tDM/cow) Grass Silage | Maize Silage | Concentrates | Calving Pattern (Number of cows) | | |
|---|---|---|---|---|---|---|
| | | Basic milk price: 18 p/litre | | | | |
| With no maize | | | | | | |
| 60 | 1.43 | 0.00 | 2.34 | Feb: 26 | Jun/Jul: 29 | Jul/Aug: 97 |
| 65 | 1.41 | 0.00 | 2.28 | Feb: 36 | | Jul/Aug: 117 |
| 70 | 1.37 | 0.00 | 2.23 | Feb: 43 | | Jul/Aug: 113 |
| With maize (10 ha) | | | | | | |
| 60 | 1.46 | 0.71 | 1.81 | Feb: 51 | Jun/Jul: 99 | |
| 65 | 1.45 | 0.71 | 1.85 | Feb: 43 | Jun/Jul: 107 | |
| 70 | 1.46 | 0.71 | 1.89 | Feb: 35 | Jun/Jul: 114 | |
| | | Basic milk price: 22 p/litre | | | | |
| With no maize | | | | | | |
| 60 | 1.43 | 0.00 | 2.34 | Feb: 26 | Jun/Jul: 29 | Jul/Aug: 97 |
| 65 | 1.41 | 0.00 | 2.28 | Feb: 36 | | Jul/Aug: 117 |
| 70 | 1.37 | 0.00 | 2.23 | Feb: 43 | | Jul/Aug: 113 |
| With maize (10 ha) | | | | | | |
| 60 | 1.46 | 0.71 | 1.81 | Feb: 51 | Jun/Jul: 99 | |
| 65 | 1.45 | 0.71 | 1.85 | Feb: 43 | Jun/Jul: 107 | |
| 70 | 1.46 | 0.71 | 1.89 | Feb: 35 | Jun/Jul: 114 | |

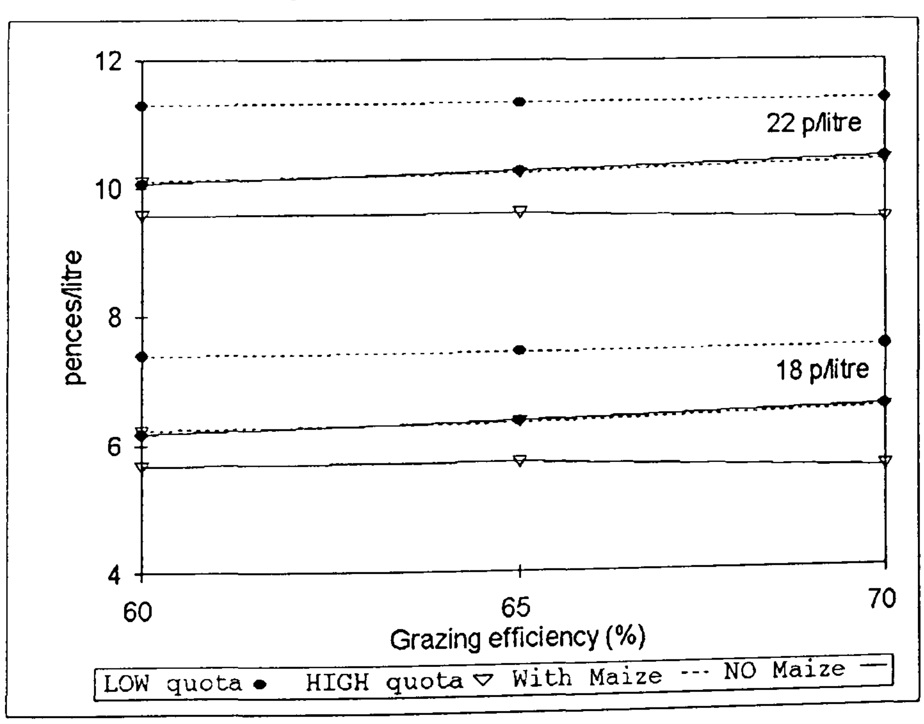
Net margins slightly increased with higher grazing efficiencies (Figure 7.29).

Figure 7.29 - Effects of grazing efficiency: net margin



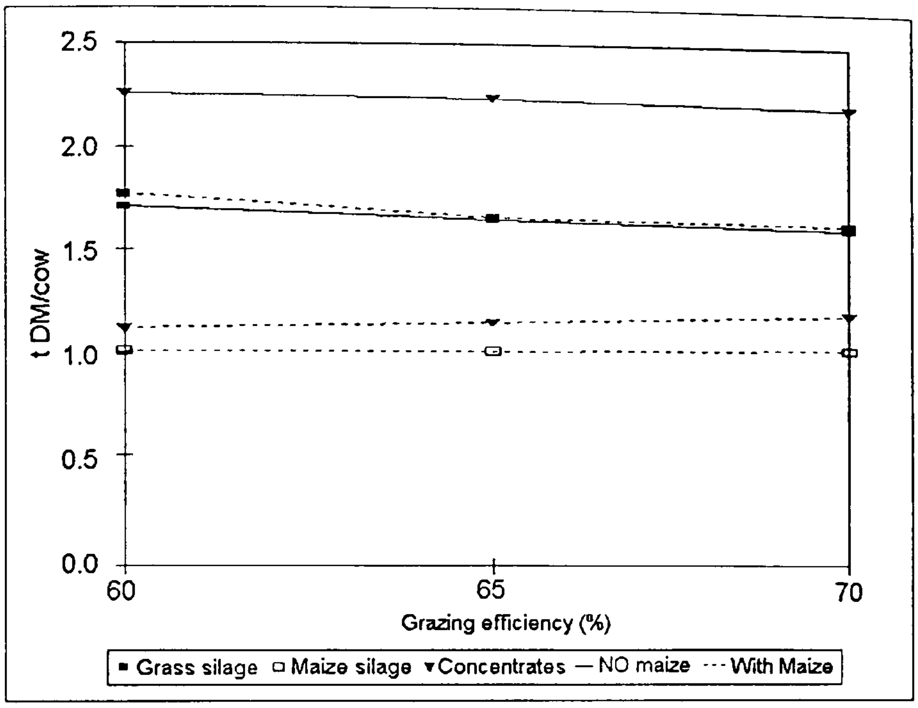

Grazing efficiency effects on marginal prices of milk quota were more evident on those scenarios with higher milk quotas and no maize, where a slight reduction of the marginal price of milk quota occurred (Figure 7.30).

Figure 7.30 - Effects of grazing efficiency: marginal price of milk quota



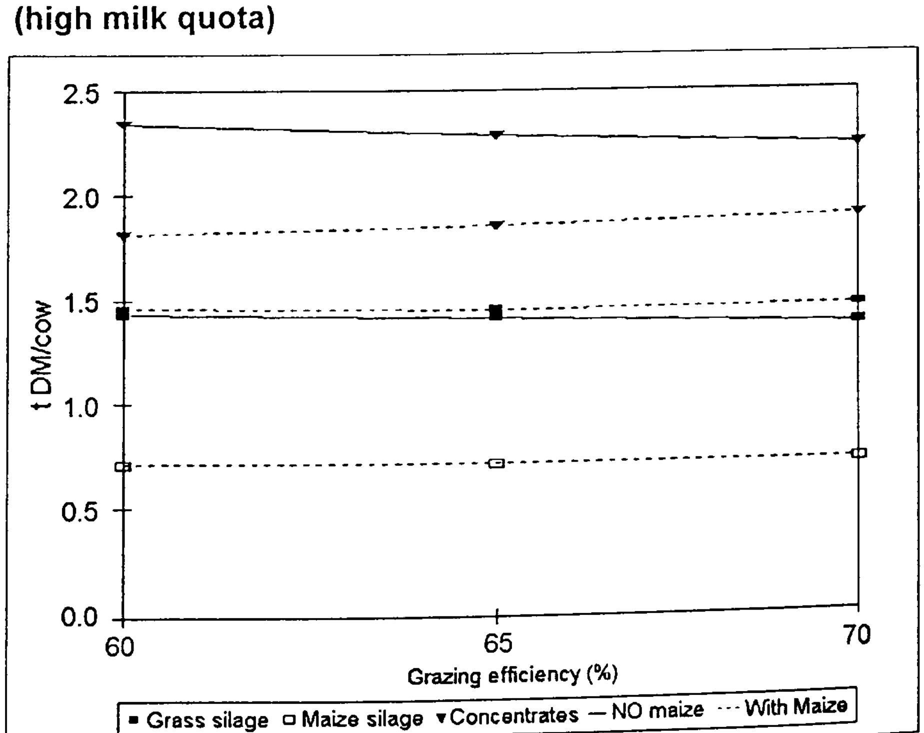

Strategic decisions were not sensitive to small differences in milk prices. When milk quota was low and there was no maize silage available, the consumption of both concentrates and grass silage were slightly reduced with higher grazing efficiencies. When maize silage was available, the consumption of concentrates was slightly increased and the consumption of grass silage slightly reduced with higher grazing efficiencies. The consumption of maize silage was not affected by the variation of grazing efficiency (Figure 7.31).

**Figure 7.31 - Effects of grazing efficiency: silage and concentrates consumption (low milk quota)**



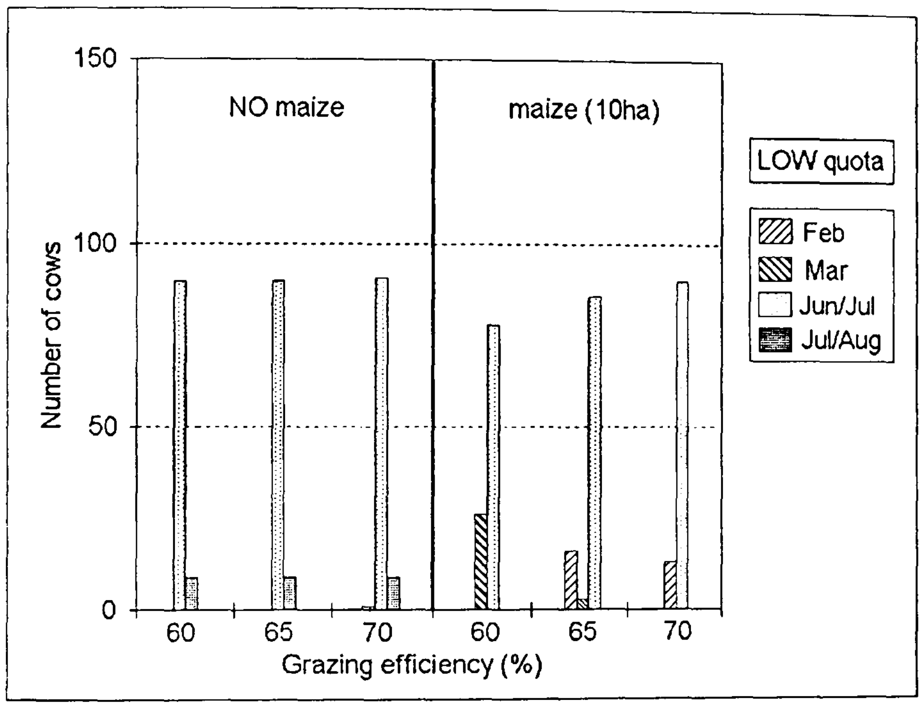

With higher milk quotas and no maize available, the consumption of concentrates and of grass silage slightly decreased with higher grazing efficiencies. When maize silage was available, grazing efficiency did not affect the consumption of maize silage or grass silage, and the consumption of concentrates slightly increased when grazing efficiency increased (Figure 7.32).

**Figure 7.32 - Effects of grazing efficiency: silage and concentrates consumption (high milk quota)**



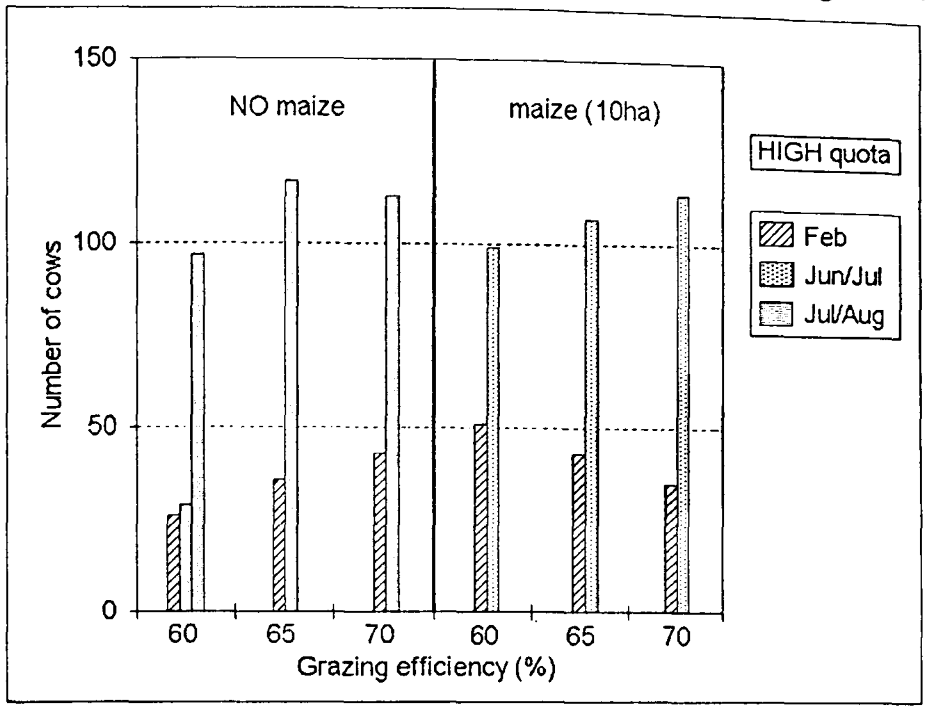
On those scenarios with lower milk quotas and no maize, the calving pattern did not change when grazing efficiency increased, although there was an indication that for higher grazing efficiencies (above 70%), the tendency would be more cows calving in February. When maize was available, the number of cows calving in Jun/Jul increased with higher grazing efficiencies, while the number of cows calving in February and March decreased, with a shift from March to February (Figure 7.33).

**Figure 7.33 - Effects of grazing efficiency: calving pattern (low milk quota)**



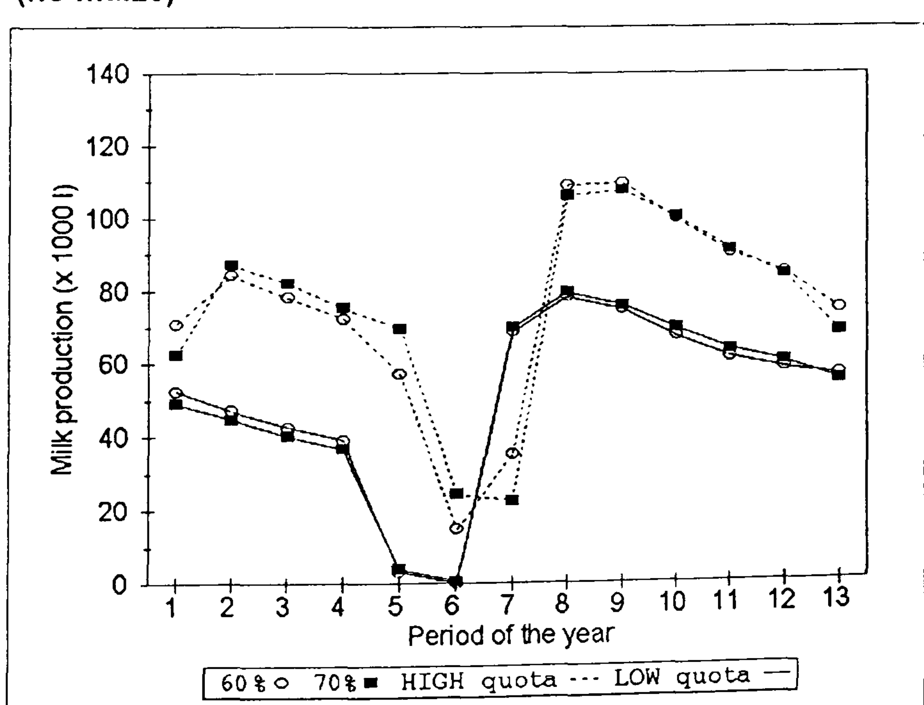

On those scenarios with higher milk quotas and no maize, when grazing efficiency increased the calving pattern was slightly changed from Summer (Jun/Jul/Aug) to February. When maize silage was available, the change was in the opposite direction, with more cows calving in Jun/Jul and less cows calving in February, when grazing efficiency increased (Figure 7.34).

**Figure 7.34 - Effects of grazing efficiency: calving pattern (high milk quota)**



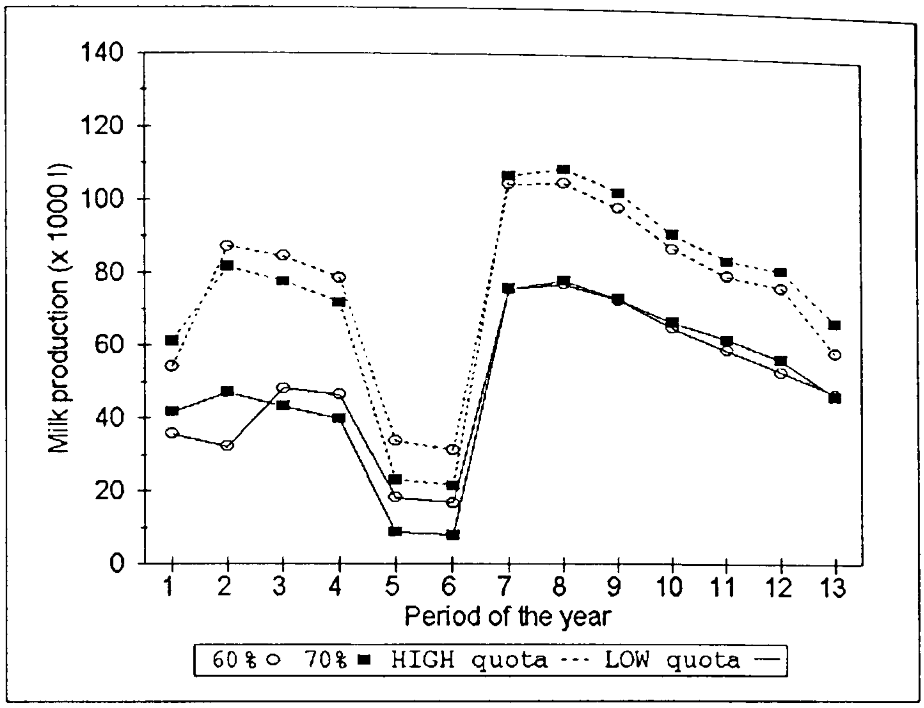

Grazing efficiency effects on total milk production throughout the year were slightly different for those scenarios with maize silage from those without maize. When maize was available, those with high quota increased their milk production in Spring, while those with lower quota produced less milk during Spring. During Summer, however, the inverse occurred (Figure 7.35).

**Figure 7.35 - Effects of grazing efficiency: total milk production over the year (no maize)**




On those scenarios with maize, high quotas and higher grazing efficiency cows produced less milk during Spring and more milk during Summer (Figure 7.36).

Figure 7.36 - Effects of grazing efficiency: total milk production over the year (with maize)




*Discussion*

The variation of grazing efficiency within the range studied did not greatly affect the strategic plans of dairy farms. When grazing efficiency increased, the calving pattern changes on those scenarios with maize showed a tendency to produce more Summer milk earlier (Jun/Jul) and to benefit from better milk prices. When maize was not available, higher grazing efficiency showed a tendency to increase Spring milk production, which meant lower prices but reduced production costs.

## 7.8. Effects of dry matter (DM) losses of grass silage

During the process of making silage there are some losses that are unavoidable, but when properly made these losses can be reduced. Use of additives can reduce silage losses and improve its digestibility, although there is a cost involved.

Dry matter losses of grass silage were estimated to be 23.8% in the standard systems. In order to study the effects of dry matter losses of grass silage on the strategic plans of dairy farms, the optimal plans are compared for farms under different conditions with low or high milk quota (630 000 and 950 000 litres) with or without maize and different DM losses of grass silage (15% and 20%).

Table 7.11 and Table 7.12 show the results predicted for farms with low and high milk quotas, respectively.

**Table 7.11 - Effects of dry matter (DM) losses of grass silage: summary of results (low milk quota)**

| DM loss grass silage (%) | Net margin (£) | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| | Basic milk price: 18 p/litre | | |
| With no maize | | | |
| 15.0 | 63 009 | 6.49 | 6286 |
| 20.0 | 61 074 | 6.28 | 6380 |
| 23.8 | 59 539 | 6.18 | 6400 |
| With maize (10 ha) | | | |
| 15.0 | 70 049 | 8.14 | 5921 |
| 20.0 | 68 659 | 7.47 | 5961 |
| 23.8 | 67 379 | 7.40 | 6030 |
| | Basic milk price: 22 p/litre | | |
| With no maize | | | |
| 15.0 | 88 198 | 10.37 | 6285 |
| 20.0 | 86 262 | 10.16 | 6380 |
| 23.8 | 84 728 | 10.06 | 6400 |
| With maize (10 ha) | | | |
| 15.0 | 95 239 | 12.02 | 5921 |
| 20.0 | 93 849 | 11.36 | 5961 |
| 23.8 | 92 569 | 11.29 | 6030 |

| DM loss grass silage (%) | Average annual consumption (tDM/cow): Grass Silage | Maize Silage | Concentrates | Calving Pattern (Number of cows) | | |
|---|---|---|---|---|---|---|
| | | | Basic milk price: 18 p/litre | | | |
| With no maize | | | | | | |
| 15.0 | 2.08 | 0.00 | 1.85 | Feb: 3 | Jun/Jul: 46 | Jul/Aug: 52 |
| 20.0 | 1.94 | 0.00 | 2.05 | | Jun/Jul: 54 | Jul/Aug: 44 |
| 23.8 | 1.71 | 0.00 | 2.26 | | Jun/Jul: 90 | Jul/Aug: 9 |
| With maize (10 ha) | | | | | | |
| 15.0 | 1.95 | 0.99 | 0.89 | Mar: 28 | Jun/Jul: 78 | |
| 20.0 | 1.87 | 1.00 | 1.01 | Mar: 27 | Jun/Jul: 78 | |
| 23.8 | 1.77 | 1.01 | 1.12 | Mar: 26 | Jun/Jul: 78 | |
| | | | Basic milk price: 22 p/litre | | | |
| With no maize | | | | | | |
| 15.0 | 2.08 | 0.00 | 1.85 | Feb: 3 | Jun/Jul: 46 | Jul/Aug: 52 |
| 20.0 | 1.94 | 0.00 | 2.05 | | Jun/Jul: 54 | Jul/Aug: 44 |
| 23.8 | 1.71 | 0.00 | 2.26 | | Jun/Jul: 90 | Jul/Aug: 9 |
| With maize (10 ha) | | | | | | |
| 15.0 | 1.95 | 0.99 | 0.89 | Mar: 28 | Jun/Jul: 78 | |
| 20.0 | 1.87 | 1.00 | 1.01 | Mar: 27 | Jun/Jul: 78 | |
| 23.8 | 1.77 | 1.01 | 1.12 | Mar: 26 | Jun/Jul: 78 | |

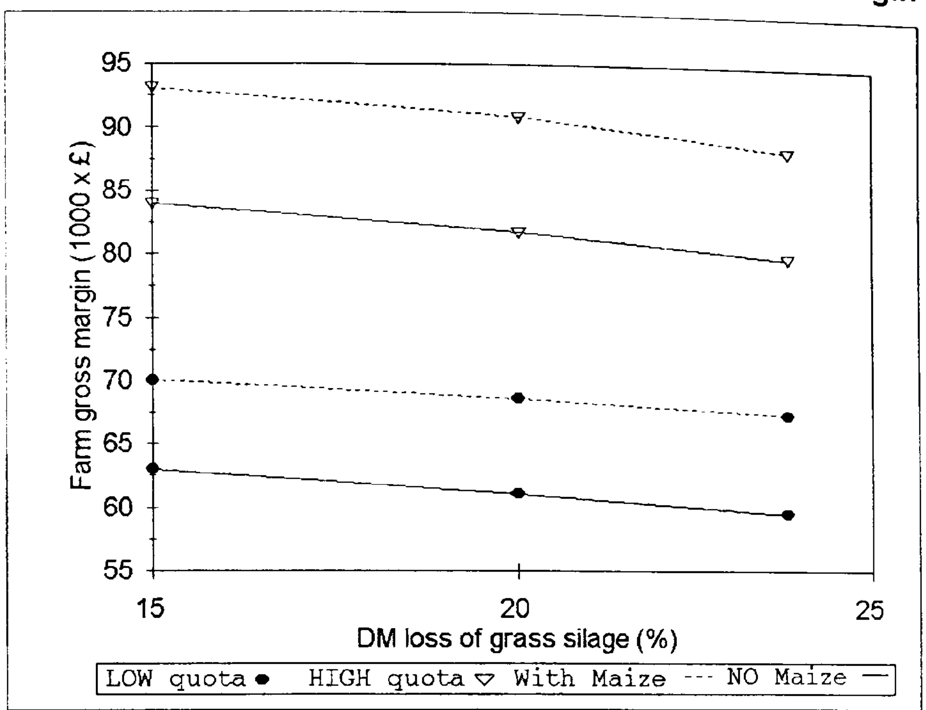
Table 7.12 - Effects of dry matter (DM) losses of grass silage: summary of results (high milk quota)

| DM loss grass silage (%) | Net margin | Marginal price of milk quota (p/litre) | Average annual milk yield (l/cow) |
|---|---|---|---|
| | Basic milk price: 18 p/litre | | |
| With no maize | | | |
| 15.0 | 83 980 | 6.11 | 6407 |
| 20.0 | 81 756 | 6.09 | 6361 |
| 23.8 | 79 763 | 5.67 | 6279 |
| With maize (10 ha) | | | |
| 15.0 | 93 141 | 6.26 | 6331 |
| 20.0 | 90 885 | 6.20 | 6371 |
| 23.8 | 88 311 | 6.23 | 6343 |
| | Basic milk price: 22 p/litre | | |
| With no maize | | | |
| 15.0 | 121 968 | 9.99 | 6407 |
| 20.0 | 119 738 | 9.97 | 6361 |
| 23.8 | 117 745 | 9.56 | 6279 |
| With maize (10 ha) | | | |
| 15.0 | 131 126 | 10.14 | 6331 |
| 20.0 | 128 870 | 10.08 | 6371 |
| 23.8 | 126 296 | 10.11 | 6343 |

| DM loss grass silage (%) | Average annual consumption (tDM/cow) Grass Silage | Maize Silage | Concentrates | Calving Pattern (Number of cows) |
|---|---|---|---|---|
| | | Basic milk price: 18 p/litre | | |
| With no maize | | | | |
| 15.0 | 1.63 | 0.00 | 2.21 | Feb: 24 Jun/Jul: 19 Jul/Aug: 105 |
| 20.0 | 1.52 | 0.00 | 2.30 | Feb: 23 Jun/Jul: 27 Jul/Aug: 99 |
| 23.8 | 1.43 | 0.00 | 2.34 | Feb: 26 Jun/Jul: 29 Jul/Aug: 97 |
| With maize (10 ha) | | | | |
| 15.0 | 1.65 | 0.70 | 1.59 | Feb: 60 Jun/Jul: 90 |
| 20.0 | 1.56 | 0.71 | 1.72 | Feb: 54 Jun/Jul: 95 |
| 23.8 | 1.46 | 0.71 | 1.81 | Feb: 51 Jun/Jul: 99 |
| | | Basic milk price: 22 p/litre | | |
| With no maize | | | | |
| 15.0 | 1.63 | 0.00 | 2.21 | Feb: 24 Jun/Jul: 19 Jul/Aug: 105 |
| 20.0 | 1.52 | 0.00 | 2.30 | Feb: 23 Jun/Jul: 27 Jul/Aug: 99 |
| 23.8 | 1.43 | 0.00 | 2.34 | Feb: 26 Jun/Jul: 29 Jul/Aug: 97 |
| With maize (10 ha) | | | | |
| 15.0 | 1.65 | 0.70 | 1.59 | Feb: 60 Jun/Jul: 90 |
| 20.0 | 1.56 | 0.71 | 1.72 | Feb: 54 Jun/Jul: 95 |
| 23.8 | 1.46 | 0.71 | 1.81 | Feb: 51 Jun/Jul: 99 |

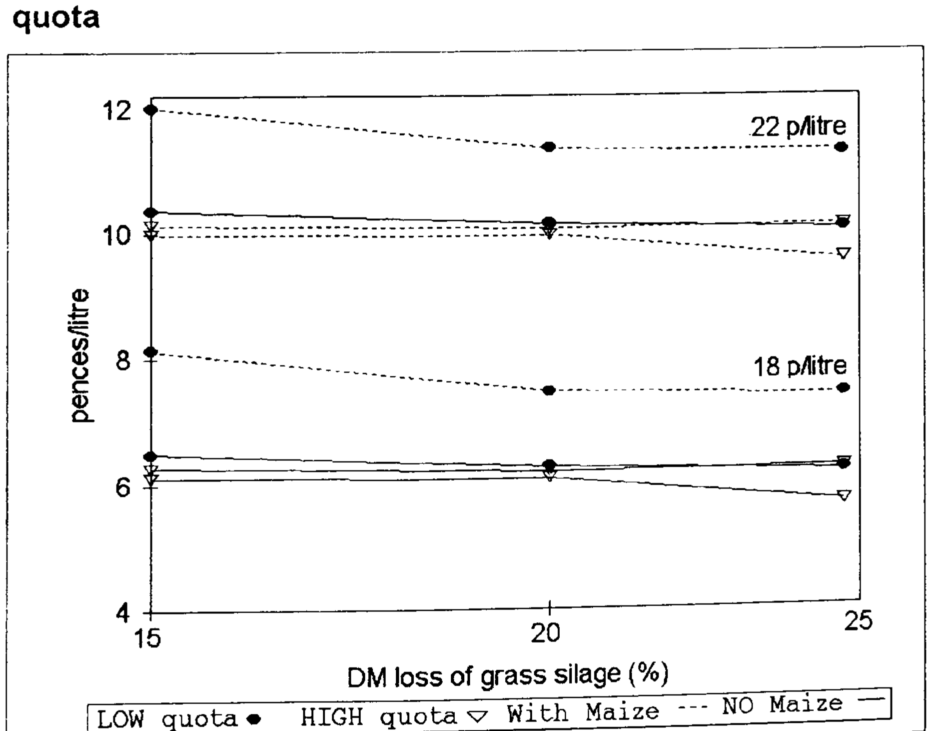
Net margins were slightly lower when losses increased (Figure 7.37).

**Figure 7.37 - Effects of dry matter (DM) losses of grass silage: net margin**




For scenarios with lower milk quota and maize silage available, marginal prices of milk quota were more reduced when losses increased from 15% to 20% than when they were above 20%. On those scenarios with higher milk quota and maize silage, the reduction of marginal prices of milk quota was enhanced when losses were above 20%. Marginal prices of milk quota were much less sensitive to different levels of DM losses of grass silage when no maize was available, although they consistently decreased as losses increased (Figure 7.38).

**Figure 7.38 - Effects of dry matter (DM) losses of grass silage: marginal price of milk quota**



The effect of DM losses of grass silage was very clear when the consumption of silage and concentrates was analysed. For scenarios with no maize silage (lower or higher milk quotas), the consumption of concentrates increased and grass silage decreased substantially when losses were higher. For those with maize silage, the consumption of concentrates also increased and grass silage also decreased, but the changes were much less substantial. Maize silage consumption was not affected by the increase of DM losses of grass silage (Figure 7.39 and Figure 7.40).

**Figure 7.39 - Effects of dry matter (DM) losses of grass silage: silage and concentrates consumption (low milk quota)**

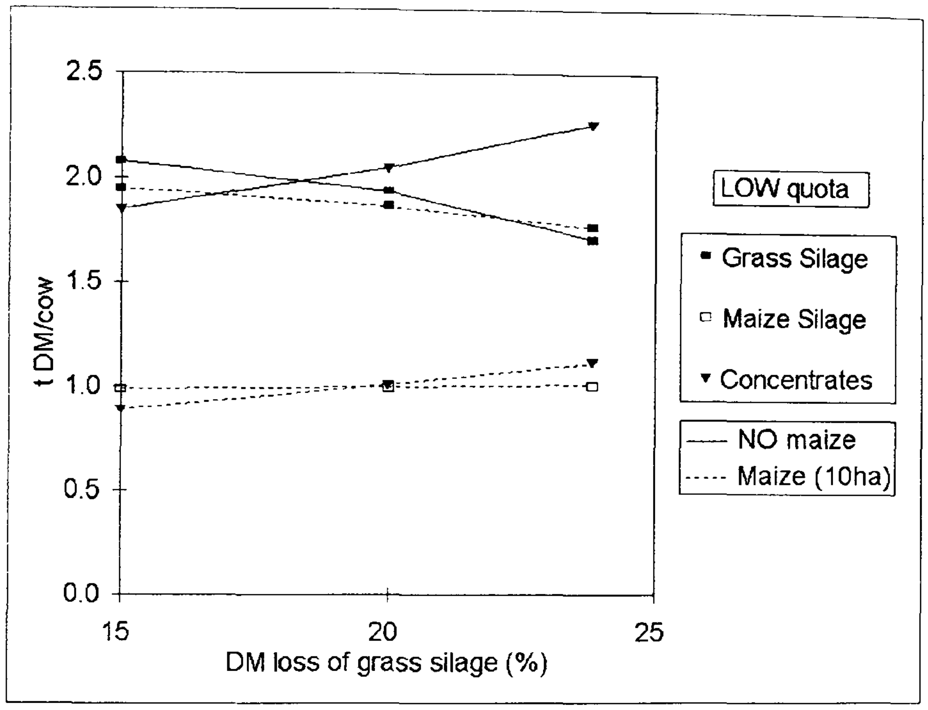
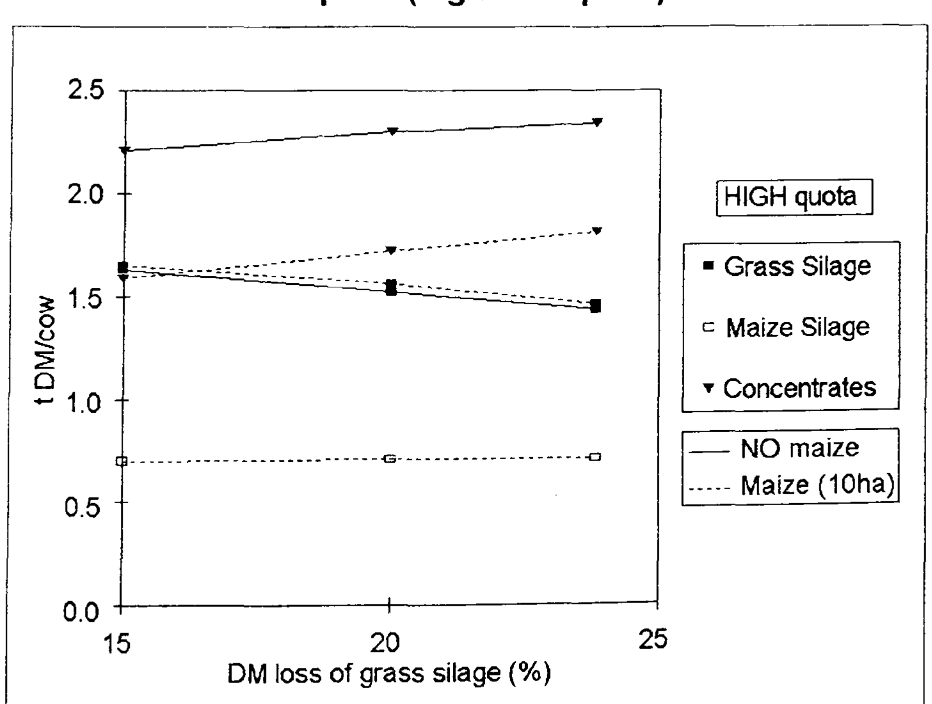
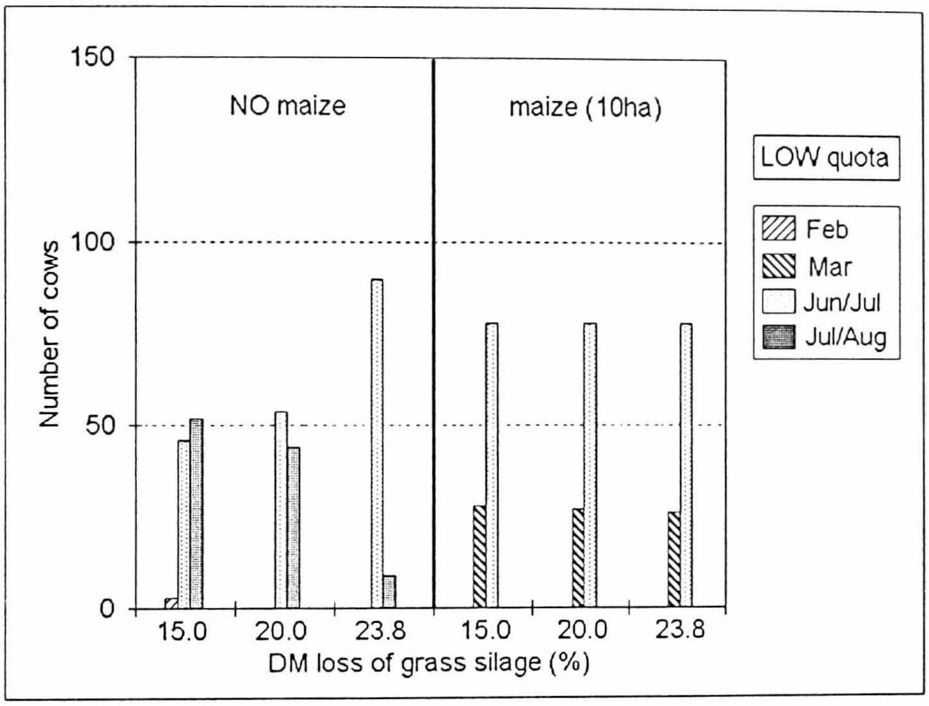



**Figure 7.40 - Effects of dry matter (DM) losses of grass silage: silage and concentrates consumption (high milk quota)**



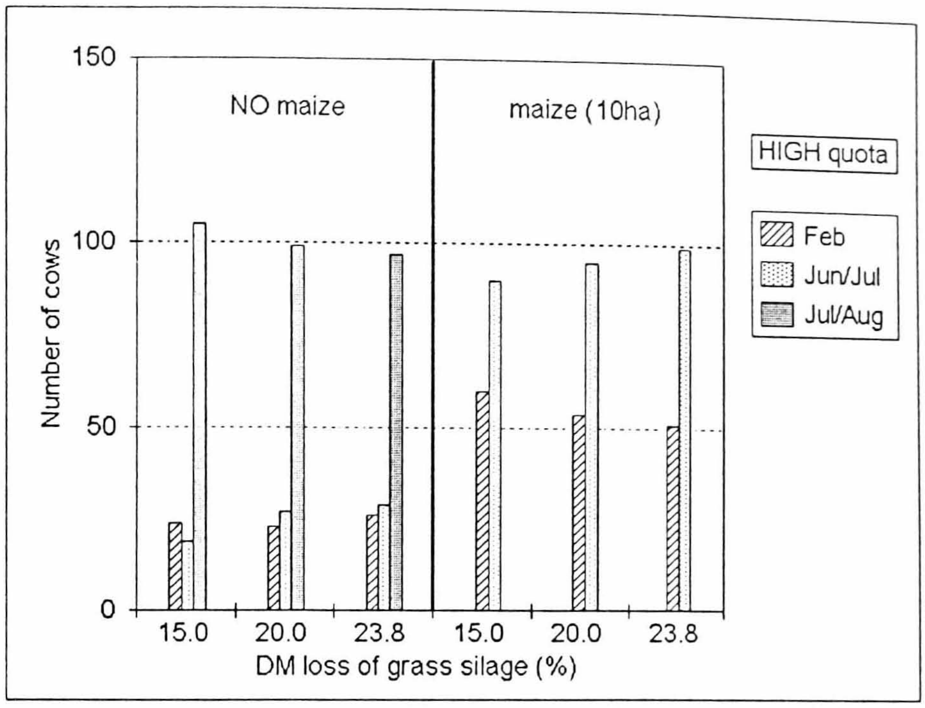
When milk quota was low, the calving pattern varied with the availability or not of maize silage. When not available, higher losses caused more cows calving in Jun/Jul and less cows calving in Jul/Aug. When maize silage was available, the number of cows calving in March was slightly reduced and the number of cows calving in Jun/Jul did not vary (Figure 7.41).

**Figure 7.41 - Effects of dry matter (DM) losses of grass silage: calving pattern (low milk quota)**




With high milk quota, even with higher losses, the number of cows calving in February was the same, while more cows calved in Jun/Jul and less cows calved in Jul/Aug on when maize silage was not available. The effect of the DM loss of grass silage, when maize silage was available, indicated a trend of changing calving from February to Jun/Jul (Figure 7.42).

**Figure 7.42 - Effects of dry matter (DM) losses of grass silage: calving pattern (high quota)**




*Discussion*

In general, DM losses of grass silage affected those scenarios where maize silage was not available. Those scenarios under those conditions are very dependent on the grass silage and changes in its quality greatly affects its consumption and the overall plan. The consumption of concentrates, which is necessary to supplement the energy requirements of cows, is also strongly affected by the quality of the silage.

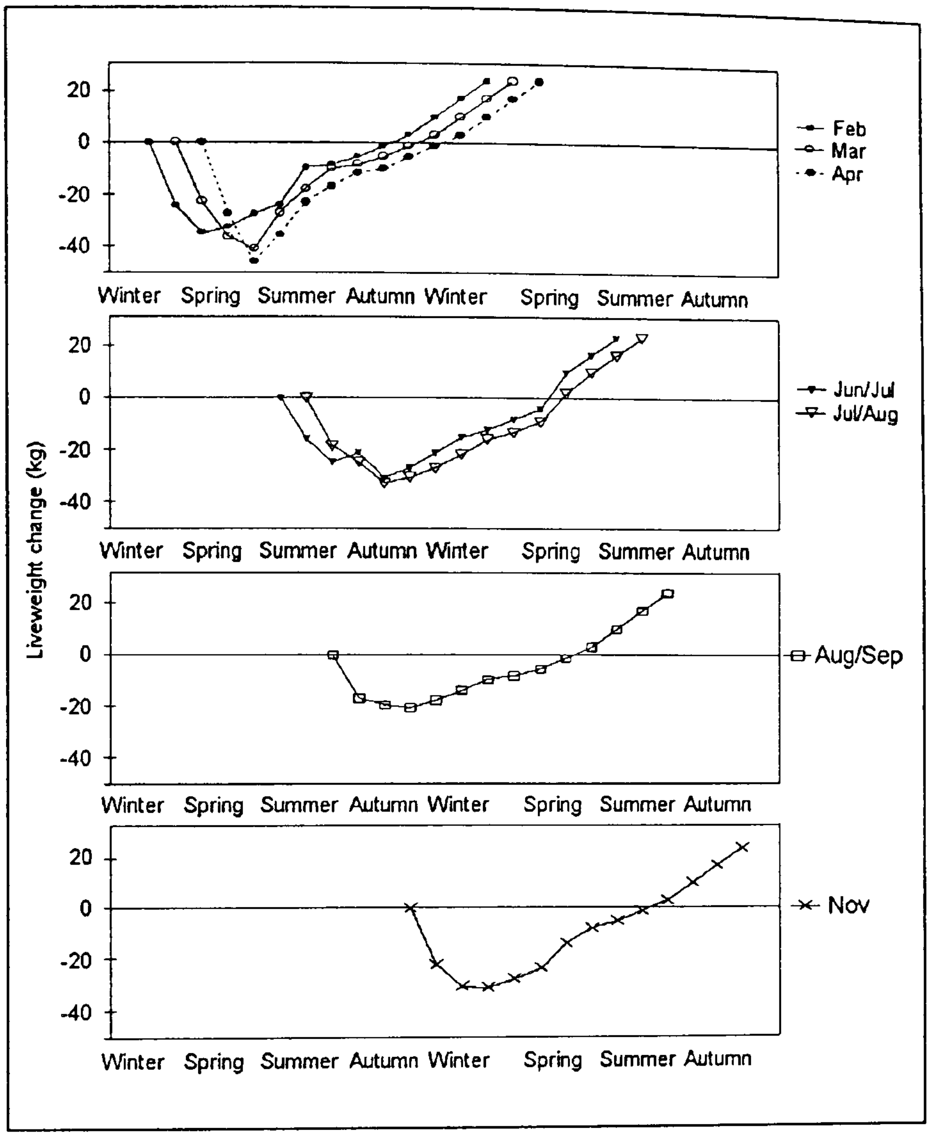
An interesting aspect of the effects of the DM losses of grass silage was the land use. In those scenarios with low quota, when losses increased, grass area decreased and cash crop area increased. With higher milk quotas, when losses increased, forage areas were about the same, with a slight reduction of cash crop area.

## 7.9. Liveweight change and milk yield patterns

Liveweight (LW) change and milk yield patterns are affected by the energy level that cows are fed. Results showed that the optimal feeding levels of cows related more to the periods in which they calved than to anything else.

Figure 7.43 shows the LW change pattern throughout the year for different calving periods which were typical of the results from different scenarios.

Figure 7.43 - Liveweight change patterns throughout the year for cows calving in different periods




Cows calving in February, March and April lost more weight than cows calving in other periods. Weight losses after parturition occurred quickly, the majority of losses also being regained quickly. They gained weight during the beginning of the grass growth season, when good quality grass is available. The optimal plans showed that it was worth feeding them to regain weight quickly in order to improve their milk production during Summer, when milk prices are higher. The later they calved the higher the weight losses and the quicker they were to regain the majority of the weight lost.

Cows calving in Jun/Jul and Jul/Aug lost less weight, but it took them a longer period to recover the weight lost. Lower LW losses after calving allowed them to have higher milk yields when the milk prices were higher. They gained weight during the Winter, when it is more expensive to feed them, and the optimal plan showed it to be worth extending the period of weight gain during this time of the year.

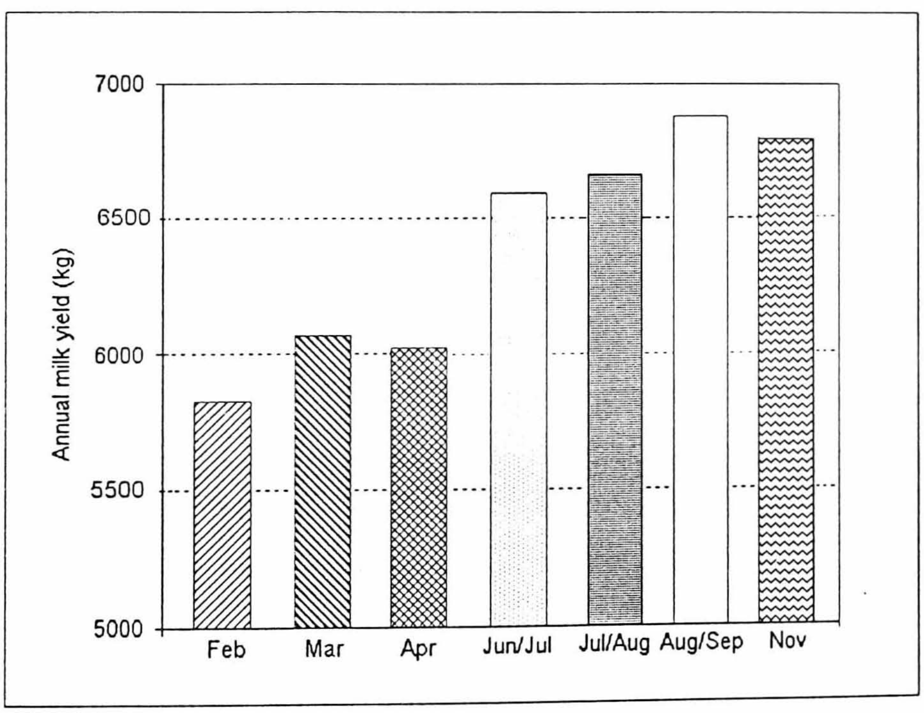
Cows calving in Aug/Sep lost even less weight after calving. The LW change curve was flat with a low weight loss and a long period to recover the majority of the weight lost.

Cows calving in this period are at the end of their lactation, and consequently with lower energy requirements during Spring. This allows larger areas for silage-making and as a consequence, the cost of feeding them during Winter will be lower. It should be noticed that cows calving in this period will be dry from the end of June until the end of August and were included in optimal plans only when milk prices were constant through the year (i.e., no seasonality of milk prices), maize silage was available and higher milk quotas were allowed.

Cows calving in November were also included in optimal plans only when milk prices were constant throughout the year, maize silage was available and farms had high milk quotas. Maize silage reduced the consumption of concentrates and consequently reduced the cost of winter feeding. During this period the cows achieved their peak milk yield and required higher levels of energy.

The average annual milk production of cows calving in different periods is shown in Figure 7.44.

**Figure 7.44 - Average annual milk yield of cows calving in different periods**

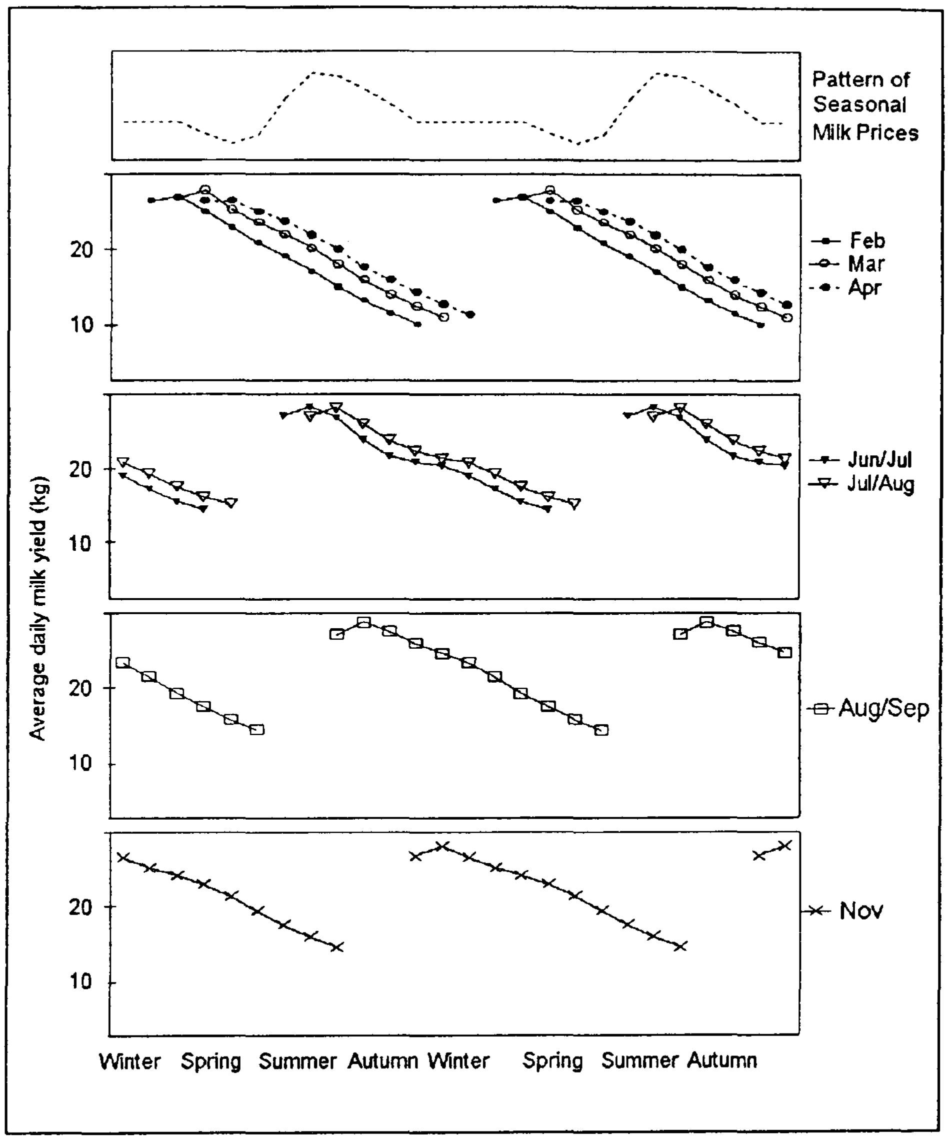



The standard annual milk yield was 5590 kg. Different milk yields were due to higher or lower feeding levels

Cows calving in February, March and April produced 5825 kg, 6070 kg, and 6020 kg, respectively. The milk yields of cows calving in Jun/Jul and Jul/Aug were 6600 kg and 6660 kg, respectively. Their annual milk yields were clearly higher (about 10% above) than that of cows producing Spring milk. Cows calving in Aug/Sep produced 6880 kg of milk over the lactation, almost 15% more than cows producing Spring milk. Annual milk yields of cows calving in November were 6800 kg, still much higher than cows calving in Spring, although slightly lower than cows calving in the Autumn.

Figure 7.45 shows the milk yield curves throughout the lactation for cows calving in different periods. It also shows the pattern of the seasonality of milk prices throughout the year.

**Figure 7.45 - Predicted milk yields throughout the year for cows calving in different periods**



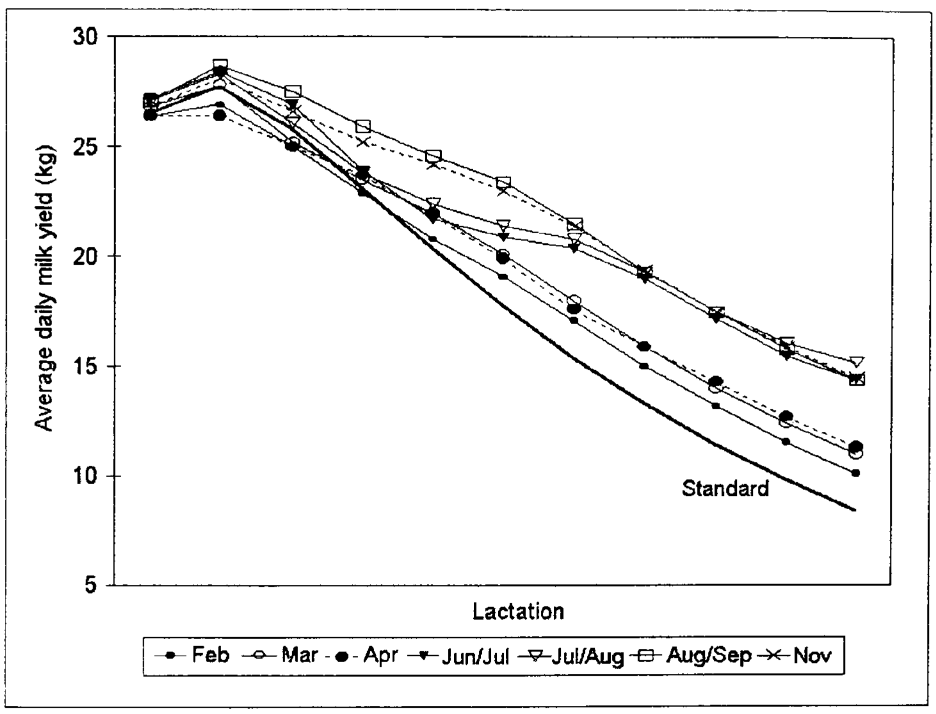

Cows calving in February, March and April achieved their peak milk yield when prices of milk were at their lowest level, during Spring. On the other hand, the feeding costs during this period were lower since the cows could graze very good quality grass. When compared to the standard milk yield (standard lactation curve), cows calving in these periods had a lower peak milk yield, although a longer period of decline of milk yield is evident. Figure 7.46 shows the milk yield over the lactation for cows calving in different periods of the year together with the standard lactation curve (see Section 4.1).

Cows calving in Jun/Jul and Jul/Aug achieved their peak milk yield when milk prices were at their highest levels during Summer. Even during their mid lactation (Winter) they were fed higher levels of energy resulting in the slope of the decline of milk yield at that stage of lactation to be lower.

Cows calving in Aug/Sep achieved their peak milk yield during Autumn. Peak yields were above the average standard and the feeding level during the subsequent stages of lactation were more uniform achieving a much higher milk yield at the end of lactation.

Cows calving in November also achieved higher peak yields than the standard, although lower than cows calving in Aug/Sep and the peak was achieved during the Winter. These cows were also fed during their mid lactation in such a way that the slope of decline of milk yield was lower, ending the lactation with a milk yield much higher than the expected.

**Figure 7.46 - Predicted milk yield throughout the lactation**




## 7.10. Summary of discussion of results

The main aspects of the results discussed in this chapter are summarized below:

- Milk quota was the major factor limiting milk production in the scenarios studied.
- When maize silage was available, it almost replaced concentrates, drastically reducing the consumption of the latter.
- Marginal prices of milk quota were higher for those scenarios with low quota and maize available. Maize silage was an important element in increasing milk production and the constraint of milk quota affected more those scenarios where maize was available. Marginal prices of milk quota were affected by the milk quota, being significantly reduced when milk quota increased to very high levels. This indicated that milk quota was an important factor limiting the production.

- Results were not sensitive to minor changes in milk prices (± 10%) in terms of the strategy adopted. Only economic figures (net margin and marginal price of milk quota) were altered when milk prices changed.
- Seasonality of milk prices particularly affected the calving pattern and consequently the milk production pattern throughout the year. When milk prices were constant throughout the year, the general trend was to have more cows calving in Spring, when good quality grass is abundant and milk production costs are lower. When milk prices varied throughout the year, the general tendency was to benefit from higher Summer milk prices by concentrating the number of calves in that period.
- The effect of change in the price of concentrates was higher for those scenarios with high milk quota and no maize. A higher production of milk was achieved by feeding higher levels and as a consequence they were much more dependent on concentrates than those with maize silage.
- When maize silage was available, there was a tendency to increase Spring milk production (more cows calving in February and March) and decrease Summer milk production (fewer cows calving in June, July and August). If the maize crop area increased, there was a tendency to produce Spring milk later (a shift from February to March calves) and a tendency to produce Summer milk earlier (shift from Jul/Aug to Jun/Jul). These shifts clearly indicate the trend of reducing costs of production and catching higher Summer milk prices, respectively.
- Cows calving in February, March and April (Spring) produced less milk over lactation than cows calving in any other period. Cows calving in June, July and August (Summer) produced substantially more milk.
- Cows calving in August/September and in November had the highest yields over the lactation. It should be remembered that cows calved in these periods only when milk prices were constant throughout the year and maize silage was available.

# Chapter Eight

## 8. Conclusions and suggestions for further work

Before proceeding with these conclusions, it should be stated that models developed to aid farm planning should be seen as assisting in decision making and not as hard and fast rules for making decisions. The purpose of such models is to provide information relevant to the decision-making process, not to make the decision. Ideally, such models should realistically represent farm systems, but in actual fact they are simplifications of the real world and results should be carefully analysed rather than just replace any previous information. For example, it is very likely that farmers have their personal preferences for taking certain actions. If the results of models suggest that the intended action is the best, then farmers may go ahead with more confidence. If the intended action is not in agreement with the results, then farmers should spend more time reconsidering their choices and examining them more deeply.

Considering the statement above, feeding level strategies which result in milk yield, weight loss and gain should not be seen as an indication of precise body energy losses or gains, but as an indication of the optimal stages for weight loss and gain.

With the previous statement in mind, some conclusions can be drawn from the results presented and discussed in Chapter 7.

### 8.1. Benefits of the model and potential applicability of the model

- The model could be used by dairy farmers (and advisors) to maximize the net margins and determine the optimal strategy for the whole farm system. Calving pattern, milk production, time to make silage, land use for grazing, maize crop or cash crop, and feeding strategy would be optimized according to the farm conditions (i.e., farm size, milk quota and labour and machinery available). Users could also make comparisons of different strategies if they change some of their options (e.g., system to make silage with related losses) or if some economic variations occur (e.g., changes of milk prices or concentrates).
- The results of the model indicated that optimal strategies for farms with maize silage are substantially different from those where maize is not available. This raises an interesting aspect that could be further investigated: whether seasonal adjustments of milk prices should be different for farms of each region of the country, according to the geographical

conditions and suitability for growing maize The model could be used by policy-makers to predict the impact of any structural change of seasonal milk prices.

- The model could also be used to analyse the impact of different fertilizer applications, which would alter the grass growth pattern (yield and quality). Different seasonal adjustments of the milk price could be applied to farms applying different levels of fertilizer. It should be remembered that the seasonality of milk price is to stimulate a spread milk production throughout the year. Since farms with different conditions have different strategies, particularly calving patterns, it seems reasonable that different milk prices could be applied and the model could be helpful for further investigations.

- The model could be used in research to study its sensitivity to some technical and biological changes. This could be helpful in determining priorities for future studies. For example, to predict the impact of a new variety of grass that could mature earlier and be available for grazing, perhaps two or three weeks earlier and whether that would cause any substantial impact on the whole system. If the impact of this new variety was not substantial, possibly another variety could be tried having a longer season perhaps ending two or three weeks later than the current variety. The impact of this could also be predicted. These analyses could be easily performed by the model, with the whole system being taken into account.

- The model could also be used, for example, to study the advantage of a new machine (or improvement of an existing one) for silage-making. Whether there would a significant effect on the system using a faster machine (with higher capacity) or a machinery that helps to reduce silage losses by reducing mechanical losses and chopping in shorter lengths. This could help to find areas where technological improvements would be more easily incorporated.

## 8.2. Limitations of the model

As already mentioned models are simplifications of reality. Specific limitations of this LP model are:

- The model proposed to describe the relationship between energy feeding levels, liveweight changes, and milk production was validated by comparing its results with data from experiments by Broster et al (1975). However, the LP is much more difficult to validate since it is not possible to find a system in practice that follows exactly that described by the model. Although the standard scenarios studied in this project were based on data from a real farm and with current prices and costs, it is not possible to make a direct evaluation. Large and complex systems such as the one described by this

LP model are usually evaluated by using representative data, checking if the optimal solution can be achieved in practice and then making sure that the results are acceptable. If this is the case, it is likely that the model closely represents the real situation, although caution is often still required in the direct application of the results from any model.

- The LP model does not take into account protein requirements. It was previously mentioned (Section 5.6) that when cows are fed large amounts of good quality grass silage, it is reasonable to assume that all their protein requirements will be provided. Since this LP model was mainly concerned with the weight change and milk yield, and these are related to the energy levels of feeding, protein requirements have not been included in the model. Unlike energy, when cows are overfed protein, it is wasted rather than being stored. Maize silage should be introduced with caution since it is high in energy but low in protein. This limitation of the model was suppressed by limiting the percentage of maize silage in the ration. If protein requirements were included in the model, this limitation could be relaxed and, possibly, the solutions could be altered with higher maize silage consumption.

- The LP model is deterministic. Risk and uncertainty are not taken into account in this model. However, several components of the model have a high level of uncertainty (e.g., feeding intake, milk production and grass growth) and when data for those elements are considered as deterministic some caution is required. Risk is created by the lack of uncertainty about the future and dairy farmers often face circumstances that create risks (e.g., weather and milk prices). Although the LP model does not take risk and uncertainty into account, it should be noticed that, by running different scenarios, it is possible to evaluate the impact that changes would cause on the decisions.

## 8.3. Suggestions for further work

These suggestions are separated into three topics according to the agricultural, mathematical and operational points of view.

- Agricultural

  - It was mentioned before that one of the limitations of the model concerns protein requirements and this subject should certainly be included in future models based on this LP. From the mathematical point of view, only minor changes will be needed, although some difficulties may arise concerning data, especially relating to crude protein (CP) and rumen degradable protein (RDP). The effects of the levels of protein on milk production can be another problem.

- Data for grass growth yield and digestibility (and related costs) were used considering one specific level of fertilizer application. Effects of different levels of fertilizer application could be included by comparing the results with different sets of data for grass growth yield and digestibility for each application level. However, some changes in the model in order to optimize the level of fertilizer application would be worthwhile, provided that there are reliable data available relating fertilizer application levels to grass yield and digestibility.

- Decisions which concern culling cows, selling or buying heifers and calves could also be included in the model. This does not seem to be complex from the mathematical point of view, although it could substantially increase the size of the problem.

- Currently, digestibility of grass (silage or grazing) indicates the energy content of the grass, estimated by a factor of conversion. Intake of these could be properly adjusted according to palatability and edibility. However, data availability could be a problem if this feature is added to the model.

- The opportunity cost of cash crops will affect the area allocated for forage and consequently the optimal feeding strategy as well as feeding and milk levels. When the opportunity cost of cash crops rises, it is expected that silage becomes less attractive relative to concentrates. It would be worth studying the effects of the opportunity cost of cash crops on the whole system.

- The current LP model ensures that any stage of lactation does not ignore information from previous and subsequent stages of lactation. This works well and it is satisfactory when the plan is for one year. However, it is well known that the feeding and milk yield levels adopted one year will also affect the cows' performance during the following years, particularly first lactation heifers. This topic seems to be important in the whole system and is worth investigation. A set of constraints transferring cows' conditions from one year to the subsequent year could be added to the model.

- Mathematical

  - The structure of the model seems to be adequate for decomposition in such a way that smaller sub-problems could be linked to one master problem. It seems to be worth investigating the feasibility of this as problems could be solved more quickly. Furthermore, this decomposition could allow an increased number of decision variables such as fields with different varieties of grass and cows of different ages. Currently, this would be impracticable to do if the model has to be solved on personal computers.

- Dynamic programming is often thought as an appropriate method for dealing with recursive problems, but it did not seem to be appropriate to be used in this case due to the particularity of the problem. Furthermore, the size of the problem would limit the efficiency of this method. However, if a decomposition of the model is done and smaller sub-problems obtained, it is worth investigating the feasibility of using dynamic programming as part of one of the sub-problems.

- Operational

  - Another interesting application is to link the LP model to a GIS (Geographical Information System), which is suitable for large databases and offers graphical display capability. Data from fields (grass, maize and cash crop yields) could be provided by the GIS and a graphical presentation of the land use then displayed.

# REFERENCES

**Agricultural Research Council (ARC). (1980).** The nutrient requirements of ruminant livestock. Commonwealth Agricultural Bureaux. Slough, UK.

**Agro Business Consultants (ABC). (1993).** Agricultural Budgeting and Costing Book No. 36. Agricultural Business Consultants Ltd, Melton Mowbray, UK.

**Audsley, E.** (1974). A linear programming model of a high-temperature grass-drying enterprise. *J. Br. Grassld Soc.* **29**: 291-298.

**Audsley, E.** (1984). A model of grass cutting and grazing for a dairy herd used to calculate the value of grass regrowth after conservation and the effect of quotas on profit. *Departmental Note DN 1241*, NIAE, Silsoe, 48 pages.

**Bastiman, B. and Altman, J.F.B.** (1985). Losses at various stages in silage making. *Research and Development in Agriculture* **2**: 19-25.

**Broster, W.H., Broster, V.J., Smith, T. and Siviter, J.W.** (1975). Experiments on the nutrition of the dairy heifer. IX. Food utilization in lactation. *J. agric. Sci., Camb.* **69**: 173-186.

**Broster, W.H., Broster, V.J. and Clements, A.J.** (1993). Feed utilization by the dairy cow over multiple lactations: a review. *Livest. Prod. Sci.* **34**: 1-21.

**Brown, C.A., Chandler, P.T. and Holter, J.B.** (1977). Development of predictive equations for milk yield and dry matter intake in lactating cows. *J. Dairy Sci.* **60**: 1739-1754.

**Catt, W.R.** (1984). Commercial Harvesters Now. British Grassland Society Occasional *Symposium No. 17*, York, 33-42.

**Corrall, A.J., Neal, H.D.St.C. and Wilkinson, J.M.** (1983). Silage in milk production: a simulation model to study the economic impact of management decisions in the production and use of silage in a dairy enterprise. Technical Report 29, Grassland Research Institute, Hurley.

**Dash** (1993). XPRESS-MP user guide. Release 7. Dash Associates. Blisworth, UK.

**Donaldson, E.** (1984). Getting the right silage. British Grassland Society Occasional *Symposium No. 17*, York, 12-17.

**Doyle, C.J. and Edwards, C.** (1986). A model for evaluating grassland management decisions on dairy farms in the UK. *Agricultural Systems* **21**: 243-266.

**Dumont, A.G. and Boyce, D.S.** (1976). Leaf protein production and use on the farm: an economic study. *Journal of the British Grassland Society* **31**: 153-163.

**Edelsten, P.R. and Corrall, A.J.** (1979). Regression models to predict herbage production and digestibility in a non-regular sequences of cuts. *J. agric. Sci., Camb.* **92**: 575-585.

**France, J., Neal, H.D.St.C. and Marsden, S.** (1982). A dairy herd cash flow model. *Agricultural Systems* **8**: 129-142.

**Goss, S.R.** (1987). Optimising milk production under quota. Ph.D. Thesis. Department of Agriculture, Horticulture and the Environment (DAHE), Wye College, University of London, 346 pp.

**Green, J.O., Corrall, A.J. and Terry, R.A.** (1971). Grass species and varieties: relationships between stage of growth, yield and forage quality. *Technical Report No. 8, Grassland Research Institute*, Hurley, 81 pp.

**Hansen, J.P.** (1990). DAIRY - An integrated Decision Support System for planning of feed supply and land use on the dairy farm. *III International Congress for Computer Technology: Integrated Decision Support Systems in Agriculture - Successful Practical Applications.* Frankfurt a.M. - Bad Soden, Germany. 182-191.

**Hulme, D.J., Kellaway, R.C. and Booth, P.J.** (1986). The CAMDAIRY model for formulating and analyzing dairy cow rations. *Agricultural Systems* **22**, 81-108.

**Killen, L. and Keane, M.** (1978). A linear programming model of seasonality in milk production. *J. Opnl. Res. Soc.* **29**: 625-631.

**Kingshay Farming Trust.** (1994). Early maize varieties at standard and marginal sites. 18 pages. Somerset, UK.

**MAFF.** (1984). Energy allowances and feeding systems for ruminants. MAFF Reference Book 433, HMSO, London.

**Mayne, C.S. and Gordon, F.J.** (1986a). The effect of harvesting system on nutrient losses during silage making: 1. Field losses. *Grass and Forage Science* **41**: 17-26.

**Mayne, C.S. and Gordon, F.J.** (1986b). The effect of harvesting system on nutrient losses during silage making: 2. In-silo losses. *Grass and Forage Science* **41**: 341-351.

**McGechan, M.B.** (1986). Mechanization aspects of forage conservation models — A review of alternative systems and their parameters. Departmental Note SIN/462. Scottish Institute of Agricultural Engineering, Penicuik, 73 pages.

**McGechan, M.B.** (1989). A review of losses arising during conservation of grass forage. Part 1: Field losses. *J. Agric. Engng Res.* **44**: 1-21.

**McGechan, M.B.** (1990a). A review of losses arising during conservation of grass forage. Part 2: Storage losses. *J. Agric. Engng Res.* **45**: 1-30.

**McGechan, M.B.** (1990b). Operational Research study of forage conservation systems for cool, humid upland climates. Part 1: description of model. *J. Agric. Engng Res.* **45**: 117-136.

**McGechan, M.B.** (1990c). Operational Research study of forage conservation systems for cool, humid upland climates. Part 2: comparison of hay and silage systems. *J. Agric. Engng Res.* **46**: 129-145.

**Neal, H.D.St.C., Thomas, C., Cobby, J.M.** (1984). Comparison of equations for predicting voluntary intake by dairy cows. *J. Agric. Sci., Camb.* **103**: 1-10.

**Nix, J.** (1995). Farm Management Pocketbook. 25th Edition, Wye College, London.

**NRC** (1988). National Research Council, Committee on Animal Nutrition, Nutrient requirements of dairy cattle. 6th rev. ed., National Academy Press, Washington, D.C.

**O'Dogherty, M.J.** (1982). A review of research on forage chopping. J. Agric. Engng Res. **27**, 267-289.

**Olney, G.R. and Falconer, D. A.** (1985). Mathematical programming model of Western Australian Dairy Farms. Western Australian Department of Agriculture, South Perth, W.A., 61 pages.

**Olney, G.R. and Kirk, G.J.** (1989). A management model that helps increase profit on Western Australian dairy farms. *Agricultural Systems* **31**, 367-380.

**Olney, G.R. and Standing, W.** (1989). Increasing profit on dairy farms. *Western Australian Journal of Agriculture* **30**: 144-149.

**Parsons, D.J.** (1992a). The Information Technology Farm 2: a simple grass production model for use in management programs. *Divisional Note DN 1639*, Silsoe Research Institute, Silsoe.

**Parsons, D.J.** (1992b). The Information Technology Farm 3: a model of dairy cow nutrition. *Technical Note 001*, IED, Silsoe Research Institute, Silsoe.

**Parsons, D.J.** (1993). A program to optimize the timing of silage harvests. *Proceedings XXV CIOSTA-CIGR V Congress, Wageningen, The Netherlands*, 111-116.

**Pichard, G., Ortega, J. and Cussen, R.** (1989). Planning seasonal forage strategies in dairy farms: a linear programming approach. *XVI International Grassland Congress*, Nice, France, 1321-1322.

**Pohjonen, V.** (1975). A dynamic model for determining the optimum cutting schedule of Italian ryegrass. *Journal of the Scientific Agricultural Society of Finland* **47**, 71-137.

**Poole, D.A.** (1986). Food intake, milk production and body-weight change of milking cows given complete diets to appetite. *Anim. Prod.* **42**: 305-313.

**Reyes, A.A., Blake, R.W., Shumway, C.R. and Long, J.T.** (1981). Multistage optimization model for dairy production. *J. Dairy Sci.* **64**: 2003-2016.

**Spedding, C.R.W.** (1988). General aspects of modelling and its application in livestock production. *In:* Modelling of Livestock Production Systems, Edited by S. Korver and J. A. M. van Arendonk, Kluwer Academic Publishers, London.

**Wood, P.D.P.** (1967). Algebraic model of the lactation curve in cattle. Nature, London **216**: 164-165.

**Wood, P.D.P.** (1969). Factors affecting the shape of the lactation curve in cattle. Anim. Prod. **11**: 307-316.

**Wood, P.D.P., King, J.O.L. and Youdan, P.G.** (1980). Relationships between size, live-weight change and milk production characters in early lactation in dairy cattle. *Anim. Prod.* **31**: 143-151.

**Woodward, S.J., Pleasants, A.B. and McCall, D.G.** (1993). A simple model for optimizing rotational grazing. *Agricultural Systems* **41**, 123-155.

# Appendix I

## Experimental results (Broster et al, 1975)

| | Treatment MG | | | | Treatment M16 | | | | Treatment M12 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ | $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ | $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ |
| 1 | 146.6 | 17.0 | -5.0 | 0.0 | 146.6 | 16.5 | -1.0 | 0.0 | 146.6 | 14.5 | -1.0 | 0.0 |
| 2 | 146.6 | 20.5 | -11.0 | 0.0 | 146.6 | 19.3 | -10.0 | 0.0 | 146.6 | 17.5 | -10.0 | 0.0 |
| 3 | 146.6 | 22.0 | -3.5 | 0.0 | 146.6 | 20.7 | -4.0 | 0.0 | 146.6 | 18.8 | -4.0 | 0.0 |
| 4 | 146.6 | 22.6 | -1.5 | 0.0 | 146.6 | 21.1 | -2.0 | 0.0 | 146.6 | 19.5 | 0.0 | 3.0 |
| 5 | 176.8 | 23.0 | 0.0 | 5.0 | 146.6 | 21.2 | -1.0 | 0.0 | 146.6 | 20.3 | -1.0 | 0.0 |
| 6 | 176.8 | 23.9 | 0.0 | 0.5 | 146.6 | 21.3 | 0.0 | 1.0 | 146.6 | 20.4 | 0.0 | 2.0 |
| 7 | 176.8 | 24.1 | 0.0 | 1.5 | 146.6 | 21.1 | 0.0 | 1.0 | 146.6 | 20.3 | 0.0 | 1.0 |
| 8 | 176.8 | 23.5 | 0.0 | 1.0 | 146.6 | 21.1 | -1.5 | 0.0 | 146.6 | 20.3 | 0.0 | 2.0 |
| 9 | 146.6 | 23.0 | -1.5 | 0.0 | 146.6 | 20.9 | 0.0 | 2.0 | 146.6 | 20.0 | -1.0 | 0.0 |
| 10 | 176.8 | 23.1 | 0.0 | 5.0 | 146.6 | 20.4 | 0.0 | 3.0 | 146.6 | 19.8 | 0.0 | 2.0 |
| 11 | 176.8 | 22.9 | 0.0 | 0.5 | 146.6 | 20.1 | -2.5 | 0.0 | 146.6 | 19.6 | 0.0 | 2.0 |
| 12 | 176.8 | 22.5 | -0.0 | 1.5 | 146.6 | 20.1 | -0.5 | 0.0 | 146.6 | 19.2 | -1.0 | 0.0 |
| 13 | 146.6 | 21.5 | -5.0 | 0.0 | 146.6 | 19.9 | 0.0 | 2.5 | 116.3 | 17.9 | -2.0 | 0.0 |
| 14 | 146.6 | 20.7 | 0.0 | 1.0 | 146.6 | 19.5 | 0.0 | 1.0 | 116.3 | 16.3 | -4.5 | 0.0 |
| 15 | 146.6 | 20.3 | -1.0 | 0.0 | 146.6 | 19.2 | -0.5 | 0.0 | 116.3 | 16.1 | -0.5 | 0.0 |
| 16 | 146.6 | 20.2 | -0.5 | 0.0 | 146.6 | 19.0 | -0.5 | 0.0 | 116.3 | 16.0 | 0.0 | 0.0 |
| 17 | 116.3 | 19.7 | -4.0 | 0.0 | 116.3 | 18.0 | -4.0 | 0.0 | 116.3 | 15.7 | -3.0 | 0.0 |
| 18 | 116.3 | 17.8 | -2.5 | 1.0 | 116.3 | 17.0 | -3.0 | 0.0 | 116.3 | 15.6 | 0.0 | 0.0 |
| 19 | 116.3 | 17.2 | -3.0 | 0.0 | 116.3 | 16.4 | -3.0 | 0.0 | 116.3 | 15.5 | -1.5 | 0.0 |
| 20 | 116.3 | 17.0 | -4.0 | 0.0 | 116.3 | 16.1 | 0.0 | 0.0 | 116.3 | 15.1 | -0.5 | 0.0 |
| 21 | 116.3 | 16.6 | 0.0 | 1.5 | 116.3 | 15.9 | -4.0 | 0.0 | 116.3 | 14.9 | 0.0 | 0.0 |
| 22 | 116.3 | 16.2 | -0.5 | 0.0 | 116.3 | 15.4 | 0.0 | 0.0 | 116.3 | 14.8 | 0.0 | 1.0 |
| 23 | 116.3 | 16.0 | -2.0 | 0.0 | 116.3 | 15.0 | -1.0 | 0.0 | 116.3 | 14.4 | 0.0 | 0.0 |
| 24 | 116.3 | 15.6 | 0.0 | 1.0 | 116.3 | 14.9 | 0.0 | 2.5 | 116.3 | 14.2 | 0.0 | 1.0 |

| | Treatment M8 | | | | Treatment M4 | | | | Treatment B | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ | $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ | $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ |
| 1 | 146.6 | 17.0 | -4.0 | 0.0 | 146.6 | 15.5 | -1.0 | 0.0 | 116.3 | 15.5 | -5.0 | 0.0 |
| 2 | 146.6 | 20.2 | -8.5 | 0.0 | 146.6 | 19.3 | -10.0 | 0.0 | 116.3 | 18.0 | -11.0 | 0.0 |
| 3 | 146.6 | 22.3 | -2.5 | 0.0 | 146.6 | 20.5 | -4.0 | 0.0 | 116.3 | 18.8 | -7.0 | 0.0 |
| 4 | 146.6 | 22.8 | -4.0 | 0.0 | 146.6 | 21.1 | -2.0 | 0.0 | 116.3 | 19.0 | -2.0 | 0.0 |
| 5 | 146.6 | 22.6 | -3.5 | 0.0 | 116.3 | 21.1 | -3.0 | 0.0 | 116.3 | 19.0 | -4.0 | 0.0 |
| 6 | 146.6 | 22.3 | 0.0 | 0.0 | 116.3 | 19.6 | -6.0 | 0.0 | 116.3 | 18.5 | 0.0 | 1.0 |
| 7 | 146.6 | 22.2 | -0.5 | 0.0 | 116.3 | 19.6 | -1.0 | 0.0 | 116.3 | 18.3 | -3.0 | 0.0 |
| 8 | 146.6 | 22.0 | 0.0 | 0.0 | 116.3 | 19.0 | -1.0 | 0.0 | 116.3 | 18.0 | 0.0 | 0.0 |
| 9 | 116.3 | 20.5 | -8.0 | 0.0 | 116.3 | 19.0 | -3.0 | 0.0 | 116.3 | 17.8 | -2.0 | 0.0 |
| 10 | 116.3 | 18.9 | -2.5 | 0.0 | 116.3 | 18.4 | -2.0 | 0.0 | 116.3 | 17.4 | 0.0 | 1.0 |
| 11 | 116.3 | 18.4 | -2.5 | 0.0 | 116.3 | 17.9 | -2.0 | 0.0 | 116.3 | 17.0 | -1.0 | 0.0 |
| 12 | 116.3 | 18.0 | -2.5 | 0.0 | 116.3 | 17.8 | -4.0 | 0.0 | 116.3 | 16.8 | -2.0 | 0.0 |
| 13 | 116.3 | 17.5 | -1.5 | 0.0 | 116.3 | 17.5 | 0.0 | 2.0 | 116.3 | 16.7 | -1.0 | 0.0 |
| 14 | 116.3 | 17.3 | 0.0 | 1.0 | 116.3 | 17.2 | -4.0 | 0.0 | 116.3 | 16.3 | 0.0 | 0.0 |
| 15 | 116.3 | 17.1 | -4.5 | 0.0 | 116.3 | 16.8 | -1.0 | 0.0 | 116.3 | 16.1 | 0.0 | 0.0 |
| 16 | 116.3 | 16.8 | -0.0 | 1.5 | 116.3 | 16.4 | 0.0 | 3.0 | 116.3 | 16.0 | 0.0 | 0.0 |
| 17 | 116.3 | 16.7 | -2.0 | 0.0 | 116.3 | 16.0 | -2.0 | 0.0 | 116.3 | 15.8 | -1.0 | 0.0 |
| 18 | 116.3 | 16.5 | 0.0 | 0.0 | 116.3 | 16.0 | 0.0 | 0.5 | 116.3 | 15.7 | 0.0 | 1.0 |
| 19 | 116.3 | 16.1 | -2.0 | 0.0 | 116.3 | 15.8 | -0.5 | 0.0 | 116.3 | 15.5 | 0.0 | 1.0 |
| 20 | 116.3 | 16.0 | -1.0 | 0.0 | 116.3 | 15.4 | -1.0 | 0.0 | 116.3 | 15.1 | -2.0 | 0.0 |
| 21 | 116.3 | 16.0 | -2.0 | 0.0 | 116.3 | 15.4 | 0.0 | 0.0 | 116.3 | 14.9 | 0.0 | 0.0 |
| 22 | 116.3 | 15.5 | 0.0 | 2.0 | 116.3 | 15.0 | -3.0 | 0.0 | 116.3 | 14.5 | 0.0 | 0.0 |
| 23 | 116.3 | 15.2 | 0.0 | 1.0 | 116.3 | 14.8 | 0.0 | 2.0 | 116.3 | 14.2 | 0.0 | 1.0 |
| 24 | 116.3 | 15.0 | -2.0 | 0.0 | 116.3 | 14.5 | 0.0 | 0.0 | 116.3 | 14.0 | 0.0 | 0.0 |

# Appendix II
## Adjusted experimental LW changes

| Week | Treatment MG $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ | Treatment M16 $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ | Treatment M12 $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 131.5 | 17.0 | -1.3 | 0.0 | 131.5 | 16.5 | -0.7 | 0.0 | 131.5 | 14.5 | 0.0 | 1.3 |
| 2 | 146.6 | 20.5 | -1.6 | 0.0 | 146.6 | 19.3 | -0.2 | 0.0 | 146.6 | 17.5 | 0.0 | 1.6 |
| 3 | 146.6 | 22.0 | -3.3 | 0.0 | 146.6 | 20.7 | -1.8 | 0.0 | 146.6 | 18.8 | 0.0 | 0.3 |
| 4 | 146.6 | 22.6 | -4.0 | 0.0 | 146.6 | 21.1 | -2.3 | 0.0 | 146.6 | 19.5 | -0.4 | 0.0 |
| 5 | 176.8 | 23.0 | 0.0 | 1.9 | 146.6 | 21.2 | -2.4 | 0.0 | 146.6 | 20.3 | -1.3 | 0.0 |
| 6 | 176.8 | 23.9 | 0.0 | 1.0 | 146.6 | 21.3 | -2.5 | 0.0 | 146.6 | 20.4 | -1.4 | 0.0 |
| 7 | 176.8 | 24.1 | 0.0 | 0.8 | 146.6 | 21.1 | -2.3 | 0.0 | 146.6 | 20.3 | -1.3 | 0.0 |
| 8 | 176.8 | 23.5 | 0.0 | 1.4 | 146.6 | 21.1 | -2.3 | 0.0 | 146.6 | 20.3 | -1.3 | 0.0 |
| 9 | 146.6 | 23.0 | 0.0 | 1.9 | 146.6 | 20.9 | -2.0 | 0.0 | 146.6 | 20.0 | -1.0 | 0.0 |
| 10 | 176.8 | 23.1 | 0.0 | 1.8 | 146.6 | 20.4 | -1.4 | 0.0 | 146.6 | 19.8 | -0.7 | 0.0 |
| 11 | 176.8 | 22.9 | 0.0 | 2.0 | 146.6 | 20.1 | -1.1 | 0.0 | 146.6 | 19.6 | -0.5 | 0.0 |
| 12 | 176.8 | 22.5 | 0.0 | 2.4 | 146.6 | 20.1 | -1.1 | 0.0 | 146.6 | 19.2 | 0.0 | 0.0 |
| 13 | 146.6 | 21.5 | -2.7 | 0.0 | 146.6 | 19.9 | -0.9 | 0.0 | 131.5 | 17.9 | 0.0 | 1.1 |
| 14 | 146.6 | 20.7 | -1.8 | 0.0 | 146.6 | 19.5 | -0.4 | 0.0 | 116.3 | 16.3 | -1.0 | 0.0 |
| 15 | 146.6 | 20.3 | -1.3 | 0.0 | 146.6 | 19.2 | 0.0 | 0.0 | 116.3 | 16.1 | -0.8 | 0.0 |
| 16 | 146.6 | 20.2 | -1.2 | 0.0 | 146.6 | 19.0 | 0.0 | 0.2 | 116.3 | 16.0 | -0.7 | 0.0 |
| 17 | 131.5 | 19.7 | -0.5 | 0.0 | 131.5 | 18.0 | 0.0 | 1.0 | 116.3 | 15.7 | -0.4 | 0.0 |
| 18 | 116.3 | 17.8 | -2.4 | 0.0 | 116.3 | 17.0 | -1.6 | 0.0 | 116.3 | 15.6 | -0.3 | 0.0 |
| 19 | 116.3 | 17.2 | -1.8 | 0.0 | 116.3 | 16.4 | -1.1 | 0.0 | 116.3 | 15.5 | -0.2 | 0.0 |
| 20 | 116.3 | 17.0 | -1.6 | 0.0 | 116.3 | 16.1 | -0.8 | 0.0 | 116.3 | 15.1 | 0.0 | 0.2 |
| 21 | 116.3 | 16.6 | -1.3 | 0.0 | 116.3 | 15.9 | -0.6 | 0.0 | 116.3 | 14.9 | 0.0 | 0.3 |
| 22 | 116.3 | 16.2 | -0.9 | 0.0 | 116.3 | 15.4 | -0.1 | 0.0 | 116.3 | 14.8 | 0.0 | 0.4 |
| 23 | 116.3 | 16.0 | -0.7 | 0.0 | 116.3 | 15.0 | 0.0 | 0.2 | 116.3 | 14.4 | 0.0 | 0.7 |
| 24 | 116.3 | 15.6 | -0.3 | 0.0 | 116.3 | 14.9 | 0.0 | 0.3 | 116.3 | 14.2 | 0.0 | 0.9 |

| Week | Treatment M8 $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ | Treatment M4 $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ | Treatment B $E_f$ | y | $\Delta w^-$ | $\Delta w^+$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 131.5 | 17.0 | -1.3 | 0.0 | 131.5 | 16.5 | 0.0 | 0.4 | 116.3 | 14.5 | -0.2 | 0.0 |
| 2 | 146.6 | 20.5 | -1.2 | 0.0 | 146.6 | 19.3 | -0.2 | 0.0 | 116.3 | 17.5 | -2.6 | 0.0 |
| 3 | 146.6 | 22.0 | -3.7 | 0.0 | 146.6 | 20.7 | -1.6 | 0.0 | 116.3 | 18.8 | -3.4 | 0.0 |
| 4 | 146.6 | 22.6 | -4.2 | 0.0 | 146.6 | 21.1 | -2.3 | 0.0 | 116.3 | 19.5 | -3.6 | 0.0 |
| 5 | 146.6 | 23.0 | -4.0 | 0.0 | 131.5 | 21.2 | -1.8 | 0.0 | 116.3 | 20.3 | -3.6 | 0.0 |
| 6 | 146.6 | 23.9 | -3.7 | 0.0 | 116.3 | 21.3 | -4.1 | 0.0 | 116.3 | 20.4 | -3.1 | 0.0 |
| 7 | 146.6 | 24.1 | -3.5 | 0.0 | 116.3 | 21.1 | -4.1 | 0.0 | 116.3 | 20.3 | -2.9 | 0.0 |
| 8 | 146.6 | 23.5 | -3.3 | 0.0 | 116.3 | 21.1 | -3.6 | 0.0 | 116.3 | 20.3 | -2.6 | 0.0 |
| 9 | 131.5 | 23.0 | -1.2 | 0.0 | 116.3 | 20.9 | -3.6 | 0.0 | 116.3 | 20.0 | -2.4 | 0.0 |
| 10 | 116.3 | 23.1 | -3.5 | 0.0 | 116.3 | 20.4 | -3.0 | 0.0 | 116.3 | 19.8 | -2.0 | 0.0 |
| 11 | 116.3 | 22.9 | -3.0 | 0.0 | 116.3 | 20.1 | -2.5 | 0.0 | 116.3 | 19.6 | -1.6 | 0.0 |
| 12 | 116.3 | 22.5 | -2.6 | 0.0 | 116.3 | 20.1 | -2.4 | 0.0 | 116.3 | 19.2 | -1.4 | 0.0 |
| 13 | 116.3 | 21.5 | -2.1 | 0.0 | 116.3 | 19.9 | -2.1 | 0.0 | 116.3 | 17.9 | -1.4 | 0.0 |
| 14 | 116.3 | 20.7 | -1.9 | 0.0 | 116.3 | 19.5 | -1.8 | 0.0 | 116.3 | 16.3 | -1.0 | 0.0 |
| 15 | 116.3 | 20.3 | -1.7 | 0.0 | 116.3 | 19.2 | -1.4 | 0.0 | 116.3 | 16.1 | -0.8 | 0.0 |
| 16 | 116.3 | 20.2 | -1.4 | 0.0 | 116.3 | 19.0 | -1.1 | 0.0 | 116.3 | 16.0 | -0.7 | 0.0 |
| 17 | 116.3 | 19.7 | -1.4 | 0.0 | 116.3 | 18.0 | -0.7 | 0.0 | 116.3 | 15.7 | -0.5 | 0.0 |
| 18 | 116.3 | 17.8 | -1.2 | 0.0 | 116.3 | 17.0 | -0.7 | 0.0 | 116.3 | 15.6 | -0.4 | 0.0 |
| 19 | 116.3 | 17.2 | -0.8 | 0.0 | 116.3 | 16.4 | -0.5 | 0.0 | 116.3 | 15.5 | -0.2 | 0.0 |
| 20 | 116.3 | 17.0 | -0.7 | 0.0 | 116.3 | 16.1 | -0.1 | 0.0 | 116.3 | 15.1 | 0.0 | 0.2 |
| 21 | 116.3 | 16.6 | -0.7 | 0.0 | 116.3 | 15.9 | -0.1 | 0.0 | 116.3 | 14.9 | 0.0 | 0.3 |
| 22 | 116.3 | 16.2 | -0.2 | 0.0 | 116.3 | 15.4 | 0.0 | 0.2 | 116.3 | 14.8 | 0.0 | 0.6 |
| 23 | 116.3 | 16.0 | 0.0 | 0.1 | 116.3 | 15.0 | 0.0 | 0.4 | 116.3 | 14.4 | 0.0 | 0.9 |
| 24 | 116.3 | 15.6 | 0.0 | 0.2 | 116.3 | 14.9 | 0.0 | 0.6 | 116.3 | 14.2 | 0.0 | 1.0 |

# Appendix III

## Milk yield and accumulated LW changes predicted by the model

| | MG | | M16 | | M12 | | M8 | | M4 | | B | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Week | Milk | LW | Milk | LW | Milk | LW | Milk | LW | Milk | LW | Milk | LW |
| 1 | 16.4 | -1.1 | 16.4 | -1.1 | 16.4 | -1.1 | 16.4 | -1.1 | 16.4 | -1.1 | 16.4 | -1.5 |
| 2 | 18.8 | -1.4 | 18.8 | -1.4 | 18.8 | -1.4 | 18.8 | -1.4 | 18.8 | -1.4 | 18.5 | -4.6 |
| 3 | 20.1 | -2.8 | 20.1 | -2.8 | 20.1 | -2.8 | 20.1 | -2.8 | 20.1 | -2.8 | 19.4 | -8.6 |
| 4 | 20.8 | -5.0 | 20.8 | -5.0 | 20.8 | -5.0 | 20.8 | -5.0 | 20.8 | -5.0 | 19.8 | -13.0 |
| 5 | 21.4 | -5.2 | 21.1 | -7.5 | 21.1 | -7.5 | 21.1 | -7.5 | 20.7 | -10.6 | 19.9 | -17.4 |
| 6 | 22.1 | -3.0 | 21.3 | -10.1 | 21.3 | -10.1 | 21.3 | -10.1 | 20.5 | -15.6 | 19.8 | -21.8 |
| 7 | 22.6 | -1.3 | 21.3 | -12.8 | 21.3 | -12.8 | 21.3 | -12.8 | 20.3 | -20.4 | 19.6 | -26.0 |
| 8 | 22.9 | 0.0 | 21.2 | -15.3 | 21.2 | -15.3 | 21.2 | -15.3 | 19.9 | -24.9 | 19.4 | -30.0 |
| 9 | 23.2 | 1.1 | 21.1 | -17.8 | 21.1 | -17.8 | 20.7 | -20.9 | 19.6 | -29.0 | 19.1 | -33.6 |
| 10 | 23.4 | 1.9 | 20.9 | -20.1 | 20.9 | -20.1 | 20.2 | -25.6 | 19.2 | -32.8 | 18.8 | -37.1 |
| 11 | 23.2 | 3.6 | 20.4 | -21.6 | 20.4 | -21.6 | 19.3 | -29.5 | 18.4 | -35.8 | 18.1 | -39.8 |
| 12 | 23.1 | 5.4 | 19.9 | -22.4 | 19.5 | -25.7 | 18.4 | -32.5 | 17.7 | -38.1 | 17.4 | -41.8 |
| 13 | 22.6 | 4.4 | 19.5 | -22.8 | 18.6 | -28.9 | 17.7 | -34.8 | 17.1 | -39.8 | 16.8 | -43.2 |
| 14 | 21.8 | 1.4 | 19.1 | -22.7 | 17.8 | -31.4 | 17.0 | -36.5 | 16.5 | -41.0 | 16.2 | -44.1 |
| 15 | 21.1 | -0.8 | 18.7 | -22.3 | 17.1 | -33.1 | 16.4 | -37.6 | 16.0 | -41.6 | 15.8 | -44.6 |
| 16 | 20.4 | -2.3 | 18.0 | -24.7 | 16.5 | -34.3 | 15.9 | -38.2 | 15.5 | -41.8 | 15.3 | -44.6 |
| 17 | 19.5 | -6.5 | 17.3 | -26.6 | 16.0 | -35.0 | 15.5 | -38.3 | 15.1 | -41.6 | 14.9 | -44.3 |
| 18 | 18.6 | -9.6 | 16.6 | -27.9 | 15.5 | -35.2 | 15.1 | -38.1 | 14.7 | -41.2 | 14.6 | -43.8 |
| 19 | 17.8 | -12.0 | 16.1 | -28.6 | 15.1 | -35.1 | 14.7 | -37.7 | 14.4 | -40.5 | 14.3 | -42.9 |
| 20 | 17.1 | -13.7 | 15.6 | -28.9 | 14.7 | -34.6 | 14.4 | -36.9 | 14.1 | -39.5 | 14.0 | -41.9 |
| 21 | 16.5 | -14.8 | 15.2 | -28.9 | 14.4 | -33.9 | 14.1 | -36.0 | 13.8 | -38.4 | 13.7 | -40.7 |
| 22 | 15.9 | -15.4 | 14.8 | -28.5 | 14.1 | -33.0 | 13.8 | -34.8 | 13.6 | -37.0 | 13.5 | -39.2 |
| 23 | 15.5 | -15.6 | 14.5 | -27.8 | 13.9 | -31.9 | 13.6 | -33.4 | 13.4 | -35.5 | 13.3 | -37.7 |
| 24 | 15.1 | -15.5 | 14.2 | -27.0 | 13.6 | -30.5 | 13.4 | -31.9 | 13.2 | -33.9 | 13.1 | -35.9 |

# Appendix IV

## Original data from Grassland Research Institute (GRI)

Dry Matter yield (tDM/ha) from original data from GRI

| Week | First Growth | Regrowth 2 weeks | 3 weeks | 4 weeks | 5 weeks | 6 weeks | 7 weeks |
|---|---|---|---|---|---|---|---|
| 15 | 1.56 | — | — | — | — | — | — |
| 16 | 2.64 | — | — | — | — | — | — |
| 17 | 3.71 | 0.85 | — | — | — | — | — |
| 18 | 4.90 | 0.73 | 1.75 | — | — | — | — |
| 19 | 6.10 | 0.60 | 1.54 | 2.90 | — | — | — |
| 20 | 7.47 | 0.51 | 1.30 | 2.47 | 4.20 | — | — |
| 21 | 8.78 | 0.38 | 1.09 | 2.10 | 3.82 | 5.40 | — |
| 22 | 9.55 | 0.33 | 0.85 | 1.75 | 3.40 | 4.97 | 6.05 |
| 23 | 10.20 | 0.30 | 0.73 | 1.40 | 2.83 | 4.45 | 5.71 |
| 24 | 10.75 | 0.29 | 0.64 | 1.13 | 2.35 | 3.82 | 5.17 |
| 25 | 11.20 | 0.29 | 0.61 | 1.00 | 1.97 | 3.15 | 4.54 |
| 26 | 11.65 | 0.32 | 0.60 | 0.97 | 1.70 | 2.70 | 3.73 |
| 27 | 11.80 | 0.33 | 0.60 | 0.95 | 1.55 | 2.40 | 3.20 |
| 28 | 11.90 | 0.35 | 0.63 | 0.95 | 1.49 | 2.10 | 2.80 |
| 29 | 12.00 | 0.33 | 0.65 | 0.97 | 1.45 | 1.97 | 2.50 |
| 30 | — | 0.33 | 0.67 | 1.00 | 1.42 | 1.90 | 2.30 |
| 31 | — | 0.33 | 0.67 | 1.03 | 1.40 | 1.89 | 2.19 |
| 32 | — | 0.33 | 0.68 | 1.05 | 1.43 | 1.90 | 2.18 |
| 33 | — | 0.33 | 0.69 | 1.08 | 1.55 | 1.95 | 2.20 |
| 34 | — | 0.30 | 0.66 | 1.10 | 1.60 | 2.00 | 2.25 |
| 35 | — | 0.24 | 0.63 | 1.05 | 1.63 | 2.07 | 2.27 |
| 36 | — | 0.20 | 0.59 | 0.95 | 1.50 | 2.10 | 2.35 |
| 37 | — | 0.16 | 0.49 | 0.85 | 1.32 | 2.00 | 2.38 |
| 38 | — | 0.11 | 0.44 | 0.75 | 1.20 | 1.80 | 2.29 |
| 39 | — | 0.05 | 0.40 | 0.66 | 1.05 | 1.62 | 2.08 |
| 40 | — | 0.10 | 0.35 | 0.56 | 0.88 | 1.30 | 1.87 |

D-value (%) from original data from GRI

| Week | First Growth | Regrowth 2 weeks | 3 weeks | 4 weeks | 5 weeks | 6 weeks | 7 weeks |
|---|---|---|---|---|---|---|---|
| 15 | 75.6 | — | — | — | — | — | — |
| 16 | 74.6 | — | — | — | — | — | — |
| 17 | 73.6 | 77.6 | — | — | — | — | — |
| 18 | 70.3 | 75.3 | 74.2 | — | — | — | — |
| 19 | 66.7 | 73.0 | 71.6 | 70.3 | — | — | — |
| 20 | 63.5 | 71.6 | 69.3 | 68.5 | 66.2 | — | — |
| 21 | 61.4 | 70.1 | 67.6 | 67.0 | 65.5 | 61.9 | — |
| 22 | 64.2 | 71.9 | 69.2 | 66.0 | 64.7 | 62.2 | 57.1 |
| 23 | 62.2 | 74.7 | 72.3 | 67.0 | 64.2 | 62.2 | 56.5 |
| 24 | 60.8 | 75.8 | 72.7 | 69.2 | 64.7 | 62.2 | 56.4 |
| 25 | 59.8 | 76.1 | 75.0 | 70.3 | 67.7 | 62.4 | 56.4 |
| 26 | 58.8 | 75.2 | 74.1 | 71.3 | 67.5 | 62.2 | 56.4 |
| 27 | 58.1 | 74.0 | 73.6 | 70.7 | 67.4 | 65.0 | 56.8 |
| 28 | 57.6 | 71.6 | 71.0 | 70.4 | 67.5 | 63.4 | 58.3 |
| 29 | 57.2 | 71.0 | 69.7 | 68.5 | 67.0 | 64.0 | 57.7 |
| 30 | — | 71.4 | 69.9 | 67.5 | 65.9 | 63.8 | 59.0 |
| 31 | — | 71.5 | 70.3 | 67.8 | 65.4 | 63.4 | 59.4 |
| 32 | — | 72.6 | 71.2 | 68.0 | 65.6 | 63.5 | 59.4 |
| 33 | — | 72.8 | 71.6 | 69.0 | 65.8 | 63.5 | 59.5 |
| 34 | — | 74.4 | 71.8 | 69.6 | 67.0 | 63.9 | 60.0 |
| 35 | — | 76.1 | 73.5 | 69.5 | 67.3 | 64.7 | 60.8 |
| 36 | — | 77.6 | 75.8 | 70.7 | 67.6 | 65.0 | 61.4 |
| 37 | — | 78.5 | 76.8 | 72.2 | 68.3 | 65.5 | 62.0 |
| 38 | — | 78.8 | 76.7 | 72.8 | 69.0 | 65.5 | 62.5 |
| 39 | — | 79.0 | 77.3 | 73.3 | 69.5 | 65.7 | 62.7 |
| 40 | — | 79.5 | 77.9 | 74.2 | 70.1 | 66.2 | 63.7 |

# APPENDIX V

## Standard MPS format files

XPRESSMP accepts an LP problem as a matrix file. The data must be presented in standard Mathematical Programming System Format (known as MPS format). This format is clear, simple and unambiguous.

In this appendix, standard MPS format for LP problems is described.

## General Description of MPS Format

An MPS format file is divided into sections by header lines that contain a keyword starting at the beginning of the line. The first line is usually a NAME header, which specifies the name of the item, and the last is usually an ENDATA line, which indicates the end of the data.

Within sections, data is placed in up to six fields in fixed positions:

| Field | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Columns | 2-3 | 5-12 | 15-22 | 25-36 | 40-47 | 50-61 |

Field 1 is used for special keywords and indicators. Fields 2, 3 and 5 are usually reserved for eight-character names of variables, constraints and other entities. Fields 4 and 6 are used for numeric values, or keywords.

Any line with an asterisk (*) in Column 1 is treated as a comment.

*Eight-character Names*

The eight-character names have a fixed format and are used to specify variables, constraints and other entities. Names are not automatically justified, so blanks are treated just like other characters. For example 'COW1 ' is not the same as 'COW 1'.

No case conversion is performed, so "machine1" is different from "MACHINE1".

Any character that can be inserted in the file may be used in an eight-character name. To avoid confusion, it is sensible to use only upper case letters and digits.

*Numeric Values*

Floating point numbers may be specified in free format within the 12 character field (including embedded blanks).

## LP Matrix Format

The MPS format input is divided in sections, which must appear in the order described below. Each section is introduced by a section indicator line, which must start in Column 1.

Section: **NAME** (compulsory)

The **NAME** header indicates the beginning of the matrix. It specifies the name of the problem, and (optionally) whether the problem is to be maximised or minimised.

| Header | Field 3 | Field 5 |
|---|---|---|
| (1-4) | (15-22) | (40-47) |
| NAME | problem name | MAXIMISE or MINIMISE |

'$$$$$$$$' is the default name when a name is not supplied. The options NOOBJECT can be declared in Field 5. If field 5 is blank, a minimisation problem is assumed.

Section: **ROWS** (compulsory)

The **ROWS** section contains declarations of all the constraints in the problem with their constraint type. (The coefficients of the constraint functions are defined in the **COLUMNS** section, described later). Any constraint and objectives are specified here.

| | | |
|---|---|---|
| Section indicator line: | Header<br>(1-4)<br>ROWS | |
| Subsequent lines: | Field 1<br>(2)<br>type | Field 2<br>(5-12)<br>constraint |

Valid types are:

- N non-restraint or objective row
- G greater than or equal to (≥) constraint
- L less than or equal to (≤) constraint
- E equality (=) constraint

The row name contains eight characters and must be unique. It may not be eight blanks.

Example:

```
ROWS
 N   PROFIT
 L   AREA
```

The example above defines row PROFIT as an objective row (non-restraint) and row AREA as a less than or equal (≤) constraint.

Section: **COLUMNS** (compulsory)

In the **COLUMNS** section al the variables of the problem are declared. The coefficients of the variables in the constraints and in the objective functions are also declared in this section.

| | | | | | |
|---|---|---|---|---|---|
| Section indicator line: | Header<br>(1-7)<br>**COLUMNS** | | | | |
| Subsequent lines: | Field 2<br>(5-12)<br>variable1 | Field 3<br>(15-22)<br>constraint1 | Field 4<br>(25-36)<br>value1 | Field 5<br>(40-47)<br>constraint2 | Field 6<br>(50-61)<br>value2 |

Each line specifies the coefficients of each variable in up to two constraints. All the coefficients of a variable must be grouped together, with the column name repeated on each line. A column may not have more than one entry in any one row. Each column name must be unique, and may not be eight blanks. If field 5 is left blank, field 6 is then ignored.

Example:

```
COLUMNS
    MACHINE   LAND          12.68      COST          -25.00
```

The example above defines two coefficients for column MACHINE: 12.68 in constraint LAND and -25.00 in constraint COST.

Section: **Right-Hand Side** (compulsory)

The right hand sides of all constraints of the problem are defined in this section. For L type constraints (≤), the RHS is an upper bound on the constraint function; for G type (≥), it is a lower bound; and for E type (=), it is a lower and an upper bound. If a constraint does not appear in the RHS vector, a value 0 (zero) is assumed.

Note that an MPS format matrix may contain more than one RHS vector in the RHS section. Only one is used in defining a problem at run-time. Another may be selected as a change RHS vector for parametric programming. All the entries in one RHS vector must be grouped together. Eight blanks is an accepted RHS name.

| | | | | | |
|---|---|---|---|---|---|
| Section indicator line: | Header (1-3) RHS | | | | |
| Subsequent lines: | Field 2 (5-12) RHSname | Field 3 (15-22) constraint1 | Field 4 (25-36) value1 | Field 5 (40-47) constraint2 | Field 6 (50-61) value2 |

Up to two RHS elements may be specified in each line and if field 5 is left blank, field 6 is ignored. If all the RHS values in a problem are zero, the RHS indicator line must still be present, but no other lines are required in the RHS section.

Example:

```
RHS
    EXMPL1    CAPITAL       500.00    AREA         100.0
```

Two RHS values in the RHS vector EXMPL1 are defined in the example above: entries of 500.00 for the constraint CAPITAL and 100.0 for the constraint AREA.

Section: **BOUNDS** (optional)

Limits on the values of individual variables, if any, are defined in this section. Although constraints could be used to define such limits, it is much more efficient to use bounds. More than one bound vector may be specified, but only one is used to define the problem each time it is run. All entries for one bound vector must be specified together. Eight blanks is an acceptable bound name.

Up to two bound entries (lower and upper) may be specified for a column in a bound vector. The default lower bound is zero, and the default upper bound is plus infinity.

| | | | | |
|---|---|---|---|---|
| Section indicator line: | Header (1-6) BOUNDS | | | |
| Subsequent lines: | Field 1 (2-3) type | Field 2 (5-12) Boundname | Field 3 (15-22) variable | Field 4 (25-36) value |

Each bound is entered on a separate line. The following types are available:

UP : The variable has an upper bound given by the value in field 4; the lower bound is zero unless a LO or ML type bound is also specified

LO : The variable has a lower bound given by the value in field 4; the upper bound is plus infinity, unless an UP type bound is also specified

FX : The variable is fixed at the value specified in field 4; no other bound may be specified for the variable.

MI : The variable has a lower bound of minus infinity; the upper bound is 0 unless an UP type bound is also specified. No value is required in field 4.

PL : The variable has a lower bound of 0 and an upper bound of plus infinity; no value is required in field 4. If no bound is specified for a variable, PL is assumed by default.

FR : The variable has a lower bound of minus infinity and an upper bound of plus infinity; no value is required in field 4.

Example:

```
BOUNDS
 LO EXMPL1    MACHINE   1.0
 UP EXMPL1    MACHINE   5.0
 FR EXMPL1    CASH
```

The example above specifies that MACHINE must take a value in the range 1.0 to 5.0 ($1 \leq$ MACHINE $\leq 5$) and that CASH has no upper or lower bound.

Line **ENDATA** (compulsory)

This line indicates the end of the matrix and must be the last line of the matrix input data.

| Line | Header (1-6) |
|---|---|
| | ENDATA |

## Example of LP matrix input

*LP Matrix*

$$\text{Max } Z = 2x_1 + 3x_2$$

subject to

| | | | | | | |
|---|---|---|---|---|---|---|
| ConstrY1 | $x_1$ | + | $x_2$ | $\leq$ | 10 |
| ConstrY2 | $2x_1$ | + | $x_2$ | $\leq$ | 16 |
| ConstrY3 | $x_1$ | + | $2x_2$ | $\geq$ | 12 |

$$x_1 \geq 0, \quad 5 \leq x_2 \leq 9$$

*Standard MPS format*

```
NAME            EXAMPLE                     MAXIMISE
* Sample of a standard MPS format file of the LP matrix above
ROWS
 N  Z
 L  ConstrY1
 L  ConstrY2
 G  ConstrY3
COLUMNS
    x1          Z               2           ConstrY1        1
    x1          ConstrY2        2           ConstrY3        1
    x2          Z               3           ConstrY1        1
    x2          ConstrY2        1           ConstrY3        2
RHS
    EXMPL       ConstrY1       10           ConstrY2       16
    EXMPL       ConstrY3       12
BOUNDS
 LO EXMPL       x2              5
 UP EXMPL       x2              9
ENDATA
```

# APPENDIX VI

## DESCRIPTION OF SCENARIOS

1. Maximum maize crop area varying from 0 to 20 ha

| Filename | Description |
|---|---|
| A1 | NO MAIZE |
| A2 | Maize crop area ≤ 5 ha |
| A3 | Maize crop area ≤ 10 ha |
| A4 | Maize crop area ≤ 15 ha |
| A5 | Maize crop area ≤ 20 ha |

2. Milk quota varying from 630 000 litres to 1100 000 litres

| Filename | Description |
|---|---|
| B1 | Milk quota = 630 000 litres |
| B2 | Milk quota = 730 000 litres |
| B3 | Milk quota = 850 000 litres |
| B4 | Milk quota = 950 000 litres |
| B5 | Milk quota = 1100 000 litres |

3. Basic milk price varying from 18 to 22 p/litre

Milk quota = 630 000 litres

| Filename | Description |
|---|---|
| C1 | Basic milk price = 18.0 p/litre |
| C2 | Basic milk price = 20.3 p/litre ...................................... (same as B1) |
| C3 | Basic milk price = 22.0 p/litre |
| C4 | Basic milk price = 18.0 p/litre + Maize crop area ≤ 10 ha |
| C5 | Basic milk price = 22.0 p/litre + Maize crop area ≤ 10 ha |

Milk quota = 950 000 litres

| Filename | Description |
|---|---|
| D1 | Basic milk price = 18.0 p/litre |
| D2 | Basic milk price = 20.3 p/litre ...................................... (same as B4) |
| D3 | Basic milk price = 22.0 p/litre |
| D4 | Basic milk price = 18.0 p/litre + Maize crop area ≤ 10 ha |
| D5 | Basic milk price = 22.0 p/litre + Maize crop area ≤ 10 ha |

4. No seasonality of milk price and milk price varying from 18 to 22 p/litre

Milk quota = 630 000 litres

| Filename | Description |
|---|---|
| E1 | Milk price = 18.0 p/litre |
| E2 | Milk price = 20.0 p/litre |
| E3 | Milk price = 22.0 p/litre |
| E4 | Milk price = 18.0 p/litre + Maize crop area ≤ 10 ha |
| E5 | Milk price = 22.0 p/litre + Maize crop area ≤ 10 ha |

Milk quota = 950 000 litres

| Filename | Description |
|---|---|
| F1 | Milk price = 18.0 p/litre |
| F2 | Milk price = 20.0 p/litre |
| F3 | Milk price = 22.0 p/litre |
| F4 | Milk price = 18.0 p/litre + Maize crop area ≤ 10 ha |
| F5 | Milk price = 22.0 p/litre + Maize crop area ≤ 10 ha |

5. Price of concentrates varying from £ 140 /tDM to £ 170 /tDM

Milk quota = 630 000 litres and Basic milk price = 18 p/litre

| Filename | Description |
|---|---|
| G1 | Price of concentrates = £ 140 /tDM |
| G2 | Price of concentrates = £ 155 /tDM ............................ (same as C1) |
| G3 | Price of concentrates = £ 170 /tDM |
| G4 | Price of concentrates = £ 140 /tDM + Maize crop area ≤ 10 ha |
| G5 | Price of concentrates = £ 170 /tDM + Maize crop area ≤ 10 ha |

Milk quota = 630 000 litres and Basic milk price = 22 p/litre

| Filename | Description |
|---|---|
| H1 | Price of concentrates = £ 140 /tDM |
| H2 | Price of concentrates = £ 155 /tDM ............................ (same as C3) |
| H3 | Price of concentrates = £ 170 /tDM |
| H4 | Price of concentrates = £ 140 /tDM + Maize crop area ≤ 10 ha |
| H5 | Price of concentrates = £ 170 /tDM + Maize crop area ≤ 10 ha |

Milk quota = 950 000 litres and Basic milk price = 18 p/litre

| Filename | Description |
|---|---|
| I1 | Price of concentrates = £ 140 /tDM |
| I2 | Price of concentrates = £ 155 /tDM ............................ (same as D1) |
| I3 | Price of concentrates = £ 170 /tDM |
| I4 | Price of concentrates = £ 140 /tDM + Maize crop area ≤ 10 ha |
| I5 | Price of concentrates = £ 170 /tDM + Maize crop area ≤ 10 ha |

Milk quota = 950 000 litres and Basic milk price = 22 p/litre

| Filename | Description |
|---|---|
| J1 | Price of concentrates = £ 140 /tDM |
| J2 | Price of concentrates = £ 155 /tDM ............................ (same as D3) |
| J3 | Price of concentrates = £ 170 /tDM |
| J4 | Price of concentrates = £ 140 /tDM + Maize crop area ≤ 10 ha |
| J5 | Price of concentrates = £ 170 /tDM + Maize crop area ≤ 10 ha |

Milk quota = 950 000 litres

| Filename | Description |
|---|---|
| F1 | Milk price = 18.0 p/litre |
| F2 | Milk price = 20.0 p/litre |
| F3 | Milk price = 22.0 p/litre |
| F4 | Milk price = 18.0 p/litre + Maize crop area ≤ 10 ha |
| F5 | Milk price = 22.0 p/litre + Maize crop area ≤ 10 ha |

5. Price of concentrates varying from £ 140 /tDM to £ 170 /tDM

Milk quota = 630 000 litres and Basic milk price = 18 p/litre

| Filename | Description |
|---|---|
| G1 | Price of concentrates = £ 140 /tDM |
| G2 | Price of concentrates = £ 155 /tDM ............................ (same as C1) |
| G3 | Price of concentrates = £ 170 /tDM |
| G4 | Price of concentrates = £ 140 /tDM + Maize crop area ≤ 10 ha |
| G5 | Price of concentrates = £ 170 /tDM + Maize crop area ≤ 10 ha |

Milk quota = 630 000 litres and Basic milk price = 22 p/litre

| Filename | Description |
|---|---|
| H1 | Price of concentrates = £ 140 /tDM |
| H2 | Price of concentrates = £ 155 /tDM ............................ (same as C3) |
| H3 | Price of concentrates = £ 170 /tDM |
| H4 | Price of concentrates = £ 140 /tDM + Maize crop area ≤ 10 ha |
| H5 | Price of concentrates = £ 170 /tDM + Maize crop area ≤ 10 ha |

Milk quota = 950 000 litres and Basic milk price = 18 p/litre

| Filename | Description |
|---|---|
| I1 | Price of concentrates = £ 140 /tDM |
| I2 | Price of concentrates = £ 155 /tDM ............................ (same as D1) |
| I3 | Price of concentrates = £ 170 /tDM |
| I4 | Price of concentrates = £ 140 /tDM + Maize crop area ≤ 10 ha |
| I5 | Price of concentrates = £ 170 /tDM + Maize crop area ≤ 10 ha |

Milk quota = 950 000 litres and Basic milk price = 22 p/litre

| Filename | Description |
|---|---|
| J1 | Price of concentrates = £ 140 /tDM |
| J2 | Price of concentrates = £ 155 /tDM ............................ (same as D3) |
| J3 | Price of concentrates = £ 170 /tDM |
| J4 | Price of concentrates = £ 140 /tDM + Maize crop area ≤ 10 ha |
| J5 | Price of concentrates = £ 170 /tDM + Maize crop area ≤ 10 ha |

6. Grazing efficiency varying from 60% to 70%

Milk quota = 630 000 litres and Basic milk price = 18 p/litre

| Filename | Description |
|---|---|
| K1 | Grazing efficiency = 60% ............................................ (same as C1) |
| K2 | Grazing efficiency = 65% |
| K3 | Grazing efficiency = 70% |
| K4 | Grazing efficiency = 60% + Maize crop area ≤ 10 ha |
| K5 | Grazing efficiency = 70% + Maize crop area ≤ 10 ha |

Milk quota = 630 000 litres and Basic milk price = 22 p/litre

| Filename | Description |
|---|---|
| L1 | Grazing efficiency = 60% ............................................ (same as C3) |
| L2 | Grazing efficiency = 65% |
| L3 | Grazing efficiency = 70% |
| L4 | Grazing efficiency = 60% + Maize crop area ≤ 10 ha |
| L5 | Grazing efficiency = 70% + Maize crop area ≤ 10 ha |

Milk quota = 950 000 litres and Basic milk price = 18 p/litre

Filename Description

| Filename | Description |
|---|---|
| M1 | Grazing efficiency = 60% ............................................ (same as D1) |
| M2 | Grazing efficiency = 65% |
| M3 | Grazing efficiency = 70% |
| M4 | Grazing efficiency = 60% + Maize crop area ≤ 10 ha |
| M5 | Grazing efficiency = 70% + Maize crop area ≤ 10 ha |

Milk quota = 950 000 litres and Basic milk price = 22 p/litre

| Filename | Description |
|---|---|
| N1 | Grazing efficiency = 60% ............................................ (same as D3) |
| N2 | Grazing efficiency = 65% |
| N3 | Grazing efficiency = 70% |
| N4 | Grazing efficiency = 60% + Maize crop area ≤ 10 ha |
| N5 | Grazing efficiency = 70% + Maize crop area ≤ 10 ha |

7. DM loss of grass silage varying from 15% to 23.8 %

Milk quota = 630 000 litres and Basic milk price = 18 p/litre

| Filename | Description |
|---|---|
| P1 | DM loss = 15.0% |
| P2 | DM loss = 20.0% |
| P3 | DM loss = 23.8% ........................................................ (same as C1) |
| P4 | DM loss = 15.0% + Maize crop area ≤ 10 ha |
| P5 | DM loss = 20.0% + Maize crop area ≤ 10 ha |

Milk quota = 630 000 litres and Basic milk price = 22 p/litre

| Filename | Description |
|---|---|
| R1 | DM loss = 15.0% |
| R2 | DM loss = 20.0% |
| R3 | DM loss = 23.8% ........................................................ (same as C3) |
| R4 | DM loss = 15.0% + Maize crop area $\leq$ 10 ha |
| R5 | DM loss = 20.0% + Maize crop area $\leq$ 10 ha |

Milk quota = 950 000 litres and Basic milk price = 18 p/litre

Filename Description

| Filename | Description |
|---|---|
| S1 | DM loss = 15.0% |
| S2 | DM loss = 20.0% |
| S3 | DM loss = 23.8% ........................................................ (same as D1) |
| S4 | DM loss = 15.0% + Maize crop area $\leq$ 10 ha |
| S5 | DM loss = 20.0% + Maize crop area $\leq$ 10 ha |

Milk quota = 950 000 litres and Basic milk price = 22 p/litre

| Filename | Description |
|---|---|
| T1 | DM loss = 15.0% |
| T2 | DM loss = 20.0% |
| T3 | DM loss = 23.8% ........................................................ (same as D3) |
| T4 | DM loss = 15.0% + Maize crop area $\leq$ 10 ha |
| T5 | DM loss = 20.0% + Maize crop area $\leq$ 10 ha |

# Appendix VII (*)

## FULL RESULTS OF ALL SCENARIOS

The purpose of this Appendix is to present the full results obtained with the LP model, generated by the Report Writer program. The files are in ASCII and can be directly read from DOS.

The structure of the results presented for each scenario is as follows:

- Title
- Dimension of the LP matrix
- Farm size, herd size and stocking rate
- Gross margins
    - Farm gross margin
    - Gross margin per forage area
    - Gross margin per cow
- Shadow prices
    - Land
    - Milk quota
- Total milk production and average annual milk yield
- Cash crop and maize area
- Grass silage area: first cut and subsequent cuts
- Feed: energy content, total annual intake and average annual intake
- Calving pattern, milk production over the year and milk prices
- Average dry matter intake throughout the year for cows calving in each period
    - High quality grass silage (from first cut areas)
    - Low quality grass silage (from second and subsequent cuts)
    - Maize silage
    - Concentrates
    - Grazing
- Average metabolisable energy intake throughout the year for cows calving in each period of each feed source as above
- Average milk production throughout the year for cows calving in each period
- Average liveweight change throughout the year for cows calving in each period

---

(*) This appendix is provided in disk for microcomputers IBM-PC compatible, which is enclosed with the thesis. Files are in ASCII format. The disk is also available on request. The author's address for contact is:

Dr. Luiz Henrique A. Rodrigues (e-mail: lique@agr.unicamp.br)
Faculdade de Engenharia Agrícola
Universidade Estadual de Campinas (UNICAMP)
Cidade Universitária "Zeferino Vaz"
13084-100 Campinas SP
BRASIL